Refining the boundary between $\mathcal{P}$ and $\mathcal{NP}$ for identical machine scheduling problems with preemption and release dates

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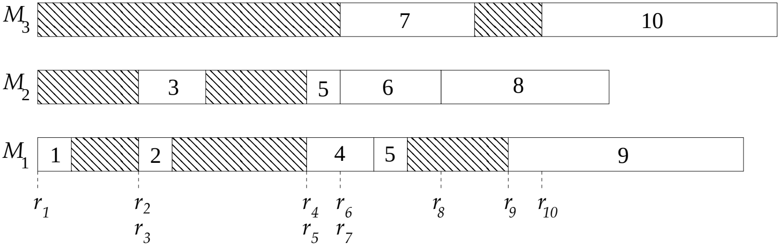
Study of the problem $P|pmtn, r_j|f$

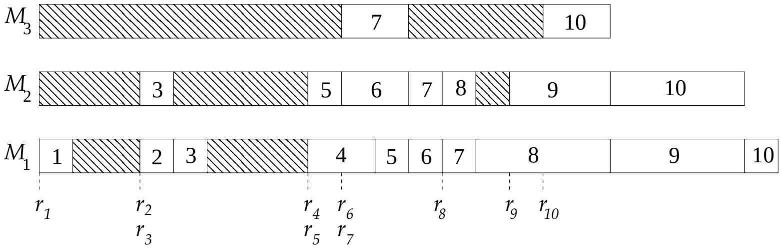
- m identical parallel machines P
- n preemptives jobs, with release date r_j such that $r_1 \leq r_2 \leq \cdots \leq r_n$
- Regular objective function f (i.e. non-decreasing with C_j's)



D. Prot, O. Bellenguez-Morineau and C. Lahlou, *Refining the boundary between P and NP for identical machine scheduling problems with preemption and release dates*, Journal of Scheduling, submitted, 2011

Definition

A schedule is said to be Permutation Flow Shop-like ($PFS - like$) if :
(1) no machine processes more than one piece of a each job,
(2) the scheduling order on the different machines is the same.
(3) it is non-delay and vertically ordered,



Theorem 1

If $P|pmtn, r_j|f$ has a solution S with completion times $C_1 \leq C_2 \leq \cdots \leq C_n$, there exists a $PFS-like$ solution S' such that $C'_1 \leq C'_2 \leq \cdots \leq C'_n$ and $C'_j \leq C_j$ for $1 \leq j \leq n$.

Proof by induction on the job number j :

- $A(j)$: If i and i' are two jobs such that $1 \leq i \leq j$ and $i < i'$ no machine processes a piece of job i' before a piece of job i.

- $B(j)$: No machine processes more than one piece of job i, for $1 \leq i \leq j$.

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Sketch of the proof - Base step

$A(1)$: no machine processes a piece of $i' > 1$ before a piece of 1.

$B(1)$: No machine processes more than one piece of job 1.

- $A(1) \implies B(1)$

- $!A(1)$: $\exists k > 1$ on M_1 before a piece of 1

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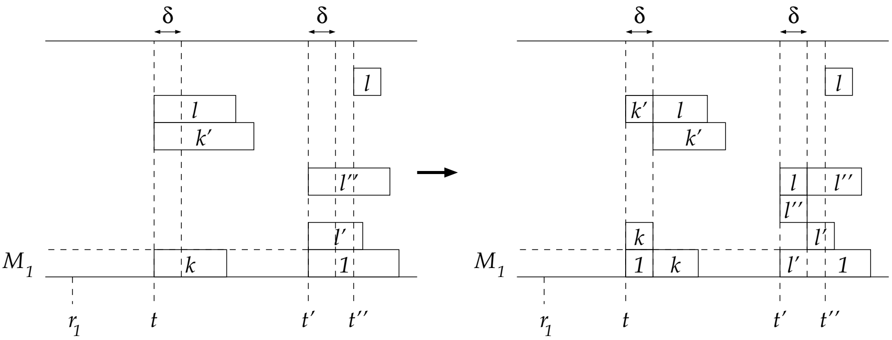

Sketch of the proof - Induction step

- $A(j) \implies B(j)$

- $!A(j)$: $\exists k > j$ on M_q before a piece of j

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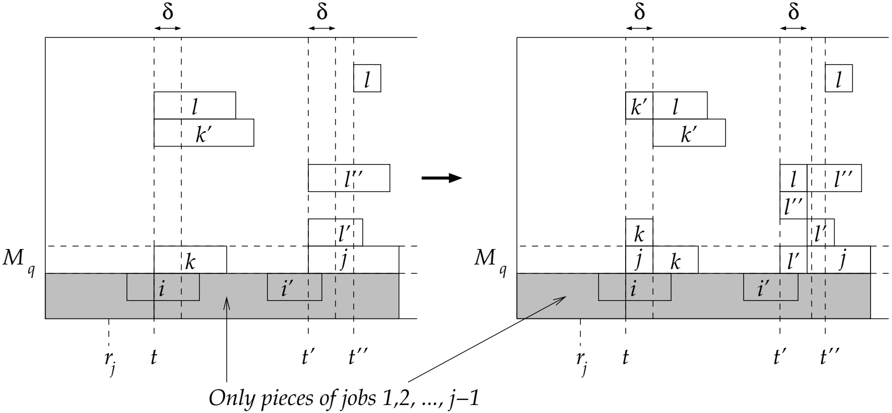

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Corollary

PFS-like schedules are dominant for $P|pmtn|f$.

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Theorem 2

$P|pmtn, r_j|f$ can be solved in polynomial time if f is a convex piecewise continuous linear function computable in polynomial time and if there exists an optimal solution such that $C_1^* \leq C_2^* \leq \cdots \leq C_n^*$.

Proof : write a LP with variables t_j^m (resp. p_j^m) for starting time (resp. processing time) of job j on machine m.

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Agreeability : $(r_j^+, p_j^+, d_j^+, w_j^-) \equiv r_j \nearrow$, $p_j \nearrow$ $d_j \nearrow$ et $w_j \searrow$
Ex : r_j,$p_j = p$,$d_j = d$, $w_j = 1$ is agreeable

Theorem 3

The following problems are solvable in polynomial time :

1. $P|pmtn, (r_j^+, p_j^+)|\sum f_j$, when f_j's are regular functions and $f_j - f_k$ is non-decreasing if $j < k$.
2. $P|pmtn, (r_j^+, p_j^+)|\max f_j$, when f_j's are regular functions and $f_j - f_k$ is non-negative if $j < k$.
3. $P|pmtn, (r_j^+, p_j^+, w_j^-), d_j = d|\sum w_j U_j$.

Proof : simple exchange argument

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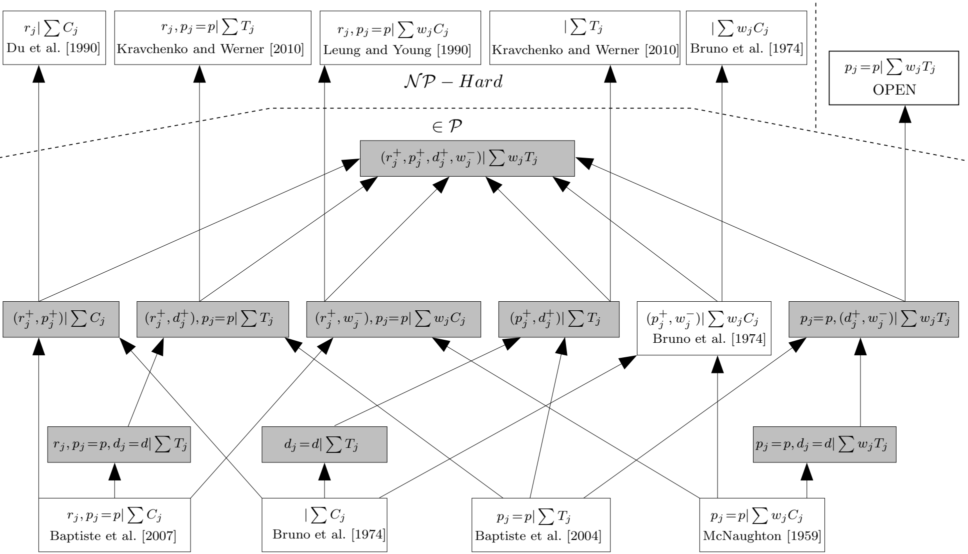
Classification of problems $P|pmtn, r_j|\sum w_j T_j$. "$P|pmtn,$" is omitted.



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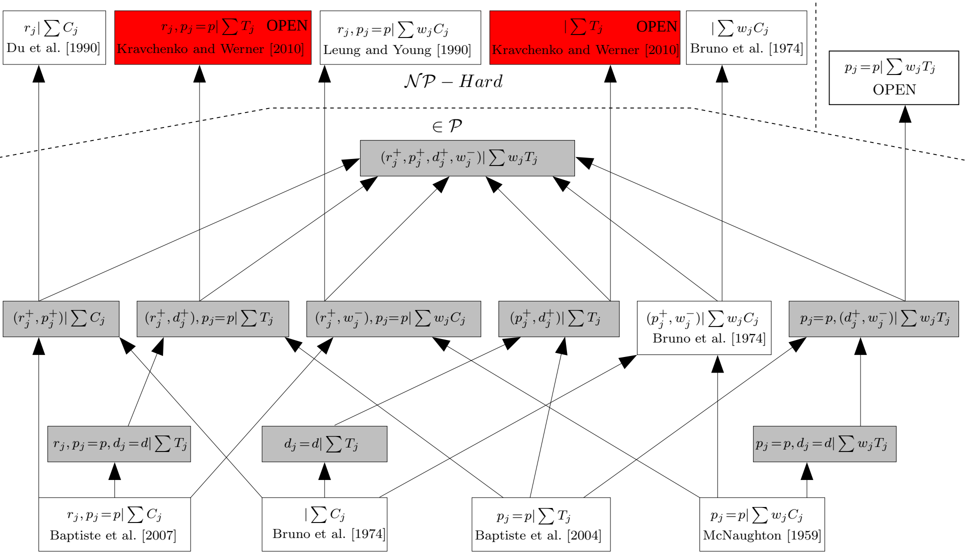

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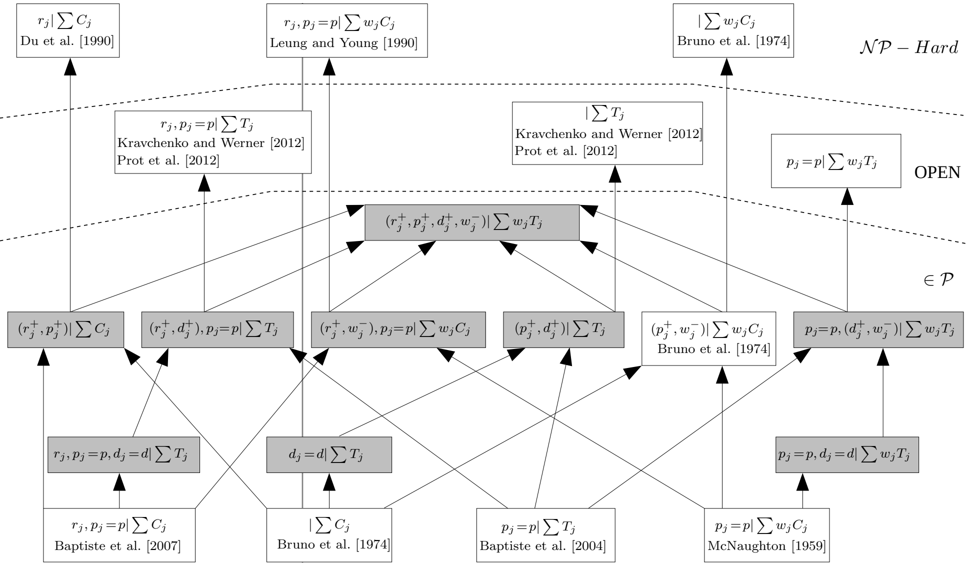

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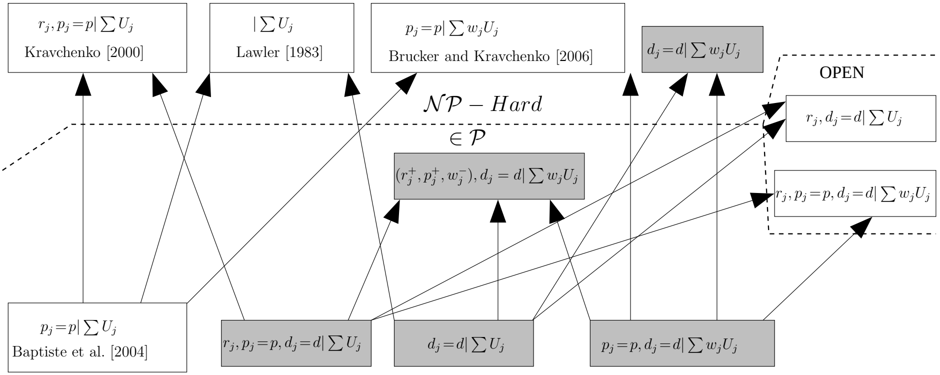

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