Single machine scheduling with delivery dates and cumulative payoffs

Yasmina Seddik, Christophe Gonzales, Safia Kedad-Sidhoum

Laboratoire d'Informatique de Paris 6, Université Pierre et Marie Curie

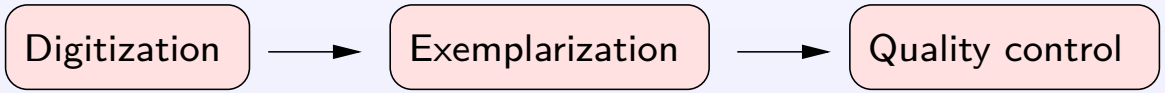
Séminaire GOThA
15 mars 2012

Outline

The industrial problem

- The Bibliothèque Nationale de France (BNF) is digitizing its entire collection.
- The digitization firm :
  - receives the books every 6 weeks,
  - handles each kind of book in a specific way (specific processing time)
  - wishes to provide at each delivery date the corresponding demanded quantity of digitized books, set by BNF

Digitization process

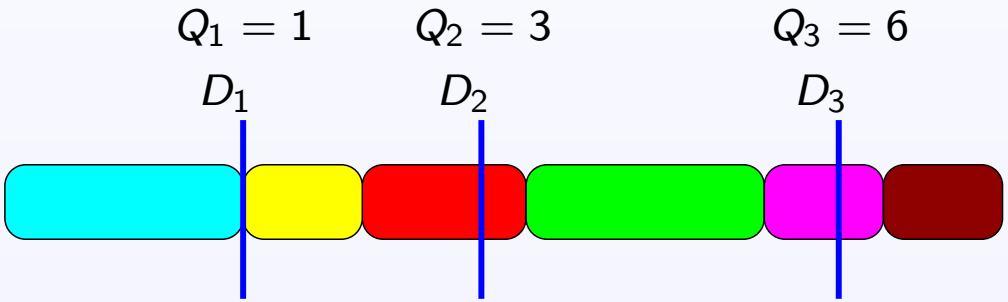


Flowshop

Problem definition (1/3) : the parameters

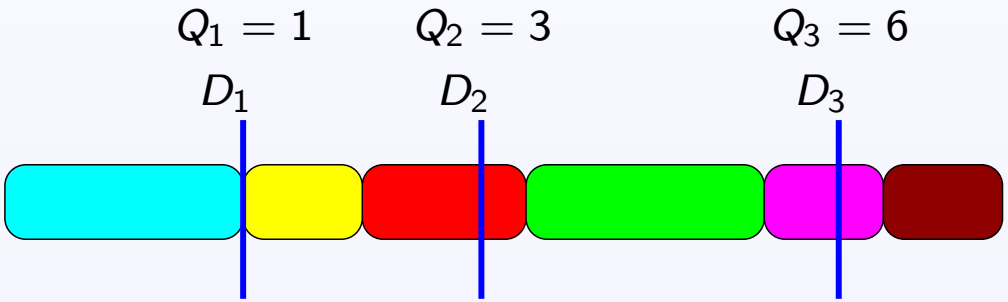
- N jobs $J_1, \ldots, J_N$
- a job J_i has :
  - a processing time p_i
  - a release date r_i
- K delivery dates $D_1, \ldots, D_K$

Problem definition (2/3) : the cumulative payoffs



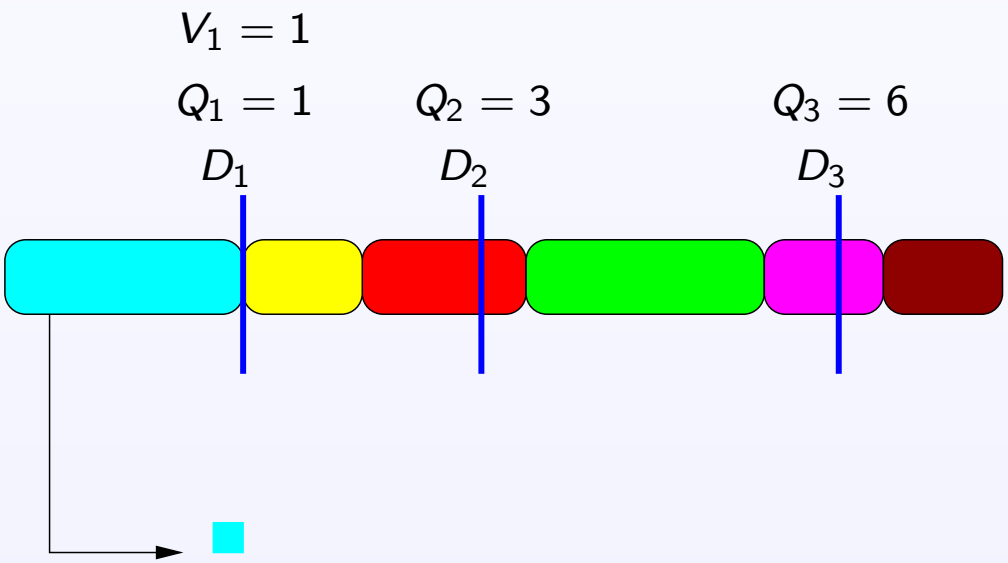
Problem definition (2/3) : the cumulative payoffs

V_k : number of jobs executed before D_k



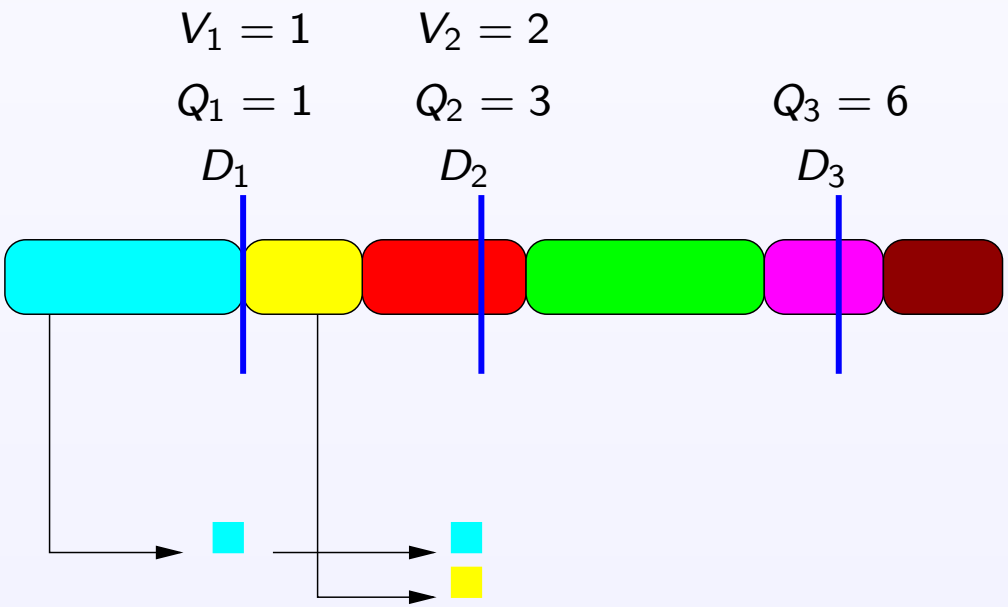
Problem definition (2/3) : the cumulative payoffs

V_k : number of jobs executed before D_k



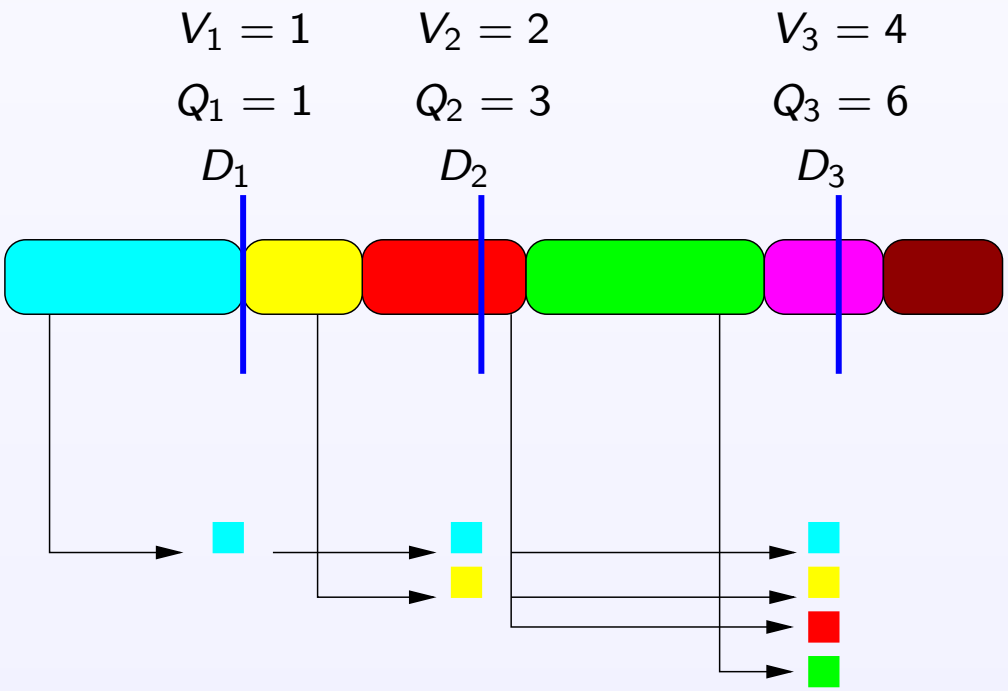
Problem definition (2/3) : the cumulative payoffs

V_k : number of jobs executed before D_k



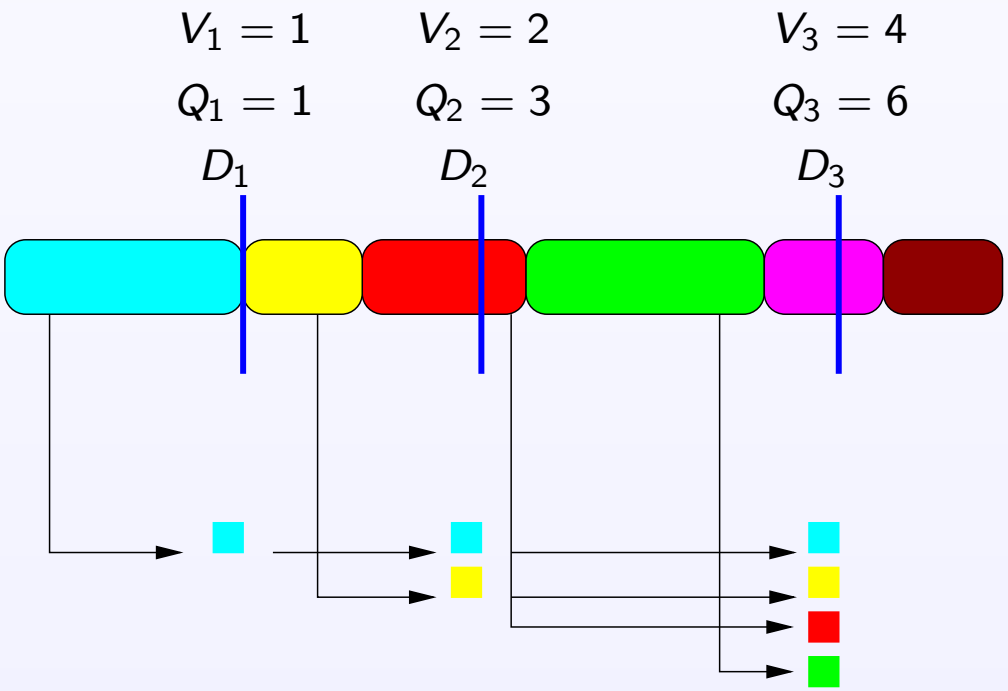
Problem definition (2/3) : the cumulative payoffs

V_k : number of jobs executed before D_k



Problem definition (2/3) : the cumulative payoffs

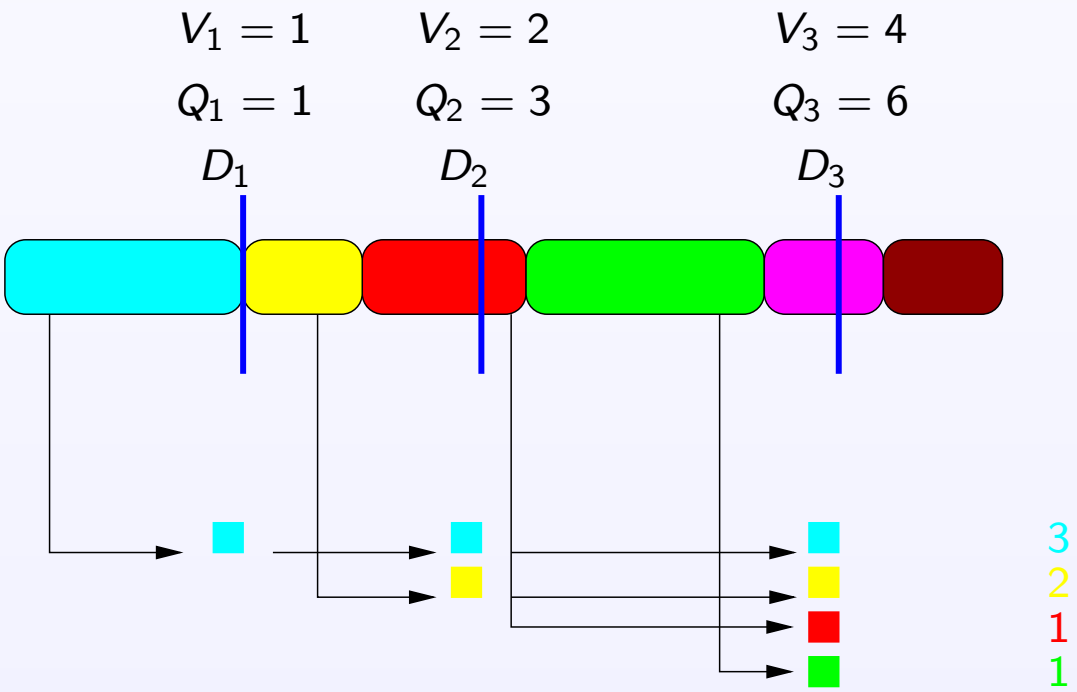
V_k : number of jobs executed before D_k



Maximizing $\sum_{k=1}^{K} V_k - Q_k \rightarrow \max \sum_{k=1}^{K} V_k$

Problem definition (2/3) : the cumulative payoffs

V_k : number of jobs executed before D_k



Maximizing $\sum_{k=1}^{K} V_k - Q_k \rightarrow \max \sum_{k=1}^{K} V_k$

Problem definition (3/3)

$$1|r_i|\sum_{k=1}^{K} V_k$$

State of the art

Without release dates:

- Unrelated parallel machines: Detienne et al., JOS (11)
- Single machine: Detienne et al.,C&OR (11) ; Tseng et al. (10)
- Single machine, common breakpoints: Yang (09)
- Single and parallel machines: Janiak and Krysiak, JOS (07)

With release dates:

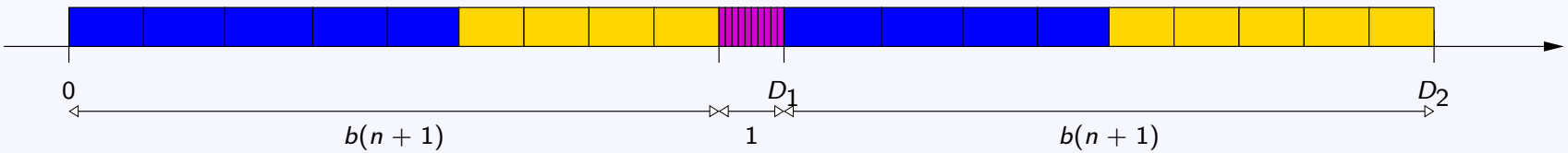
- Unrelated parallel machines: Detienne et al., JOS (11)
- Single machine: Sahin and Ahuja (11)

Online arriving jobs with rescheduling:

- Parallel machines: Curry and Peters, IJPR (05)

The two delivery dates problem

- $K = 2$
- $1|r_i|V_1 + V_2$ is weakly NP-hard.
- Reduction from Partition $(A = \{a_1, \ldots, a_n\}, \sum_{a_i \in A} a_i = 2b)$



	p_i	r_i
n jobs $\tilde{J}_i$	$b + a_i$	0
n jobs $\widehat{J}_i$	b	0
n jobs $\overline{J}_i$	$\frac{1}{n}$	$b(n+1)$

- $V = 5n$

General problem (1/2)

- The general problem is strongly NP-hard
- Reduction from 3-Partition

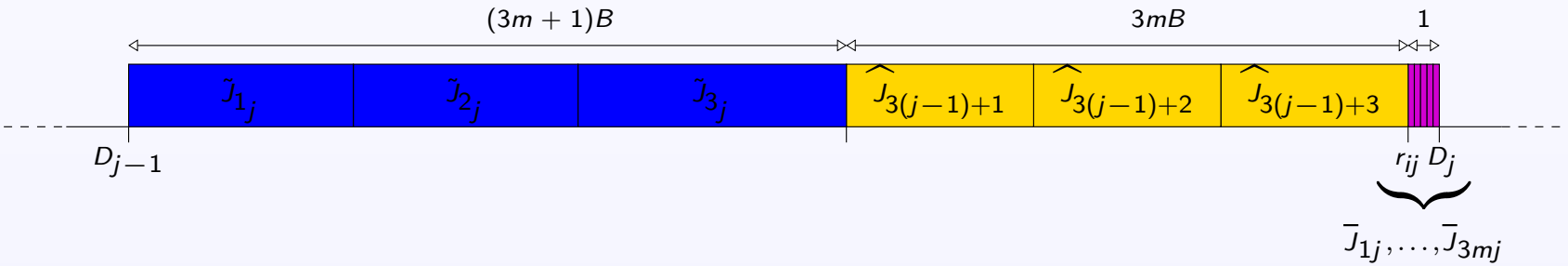
3-Partition :

- $A = \{a_1, a_2, \ldots, a_{3m}\}$ s.t. $\sum_{i=1}^{3m} a_i = mB$ and $B/4 < a_i < B/2$ for $i = 1, \ldots, 3m$
- Does there exist a partition $< A_1, A_2, \ldots, A_m >$ s.t. $\forall i, \sum_{a \in A_i} a = B$?

General problem (2/2)

Instance of $1|r_i|\sum V_k$:

- m delivery dates



	p_i	r_i
$3m$ jobs $\tilde{J}_i$	$mB + a_i$	0
$3m$ jobs $\widehat{J}_i$	mB	0
$3m^2$ jobs $\overline{J}_{ij}$	$\frac{1}{3m}$	$D_j - 1$

- $V = (6+3m)m(m+1)/2$

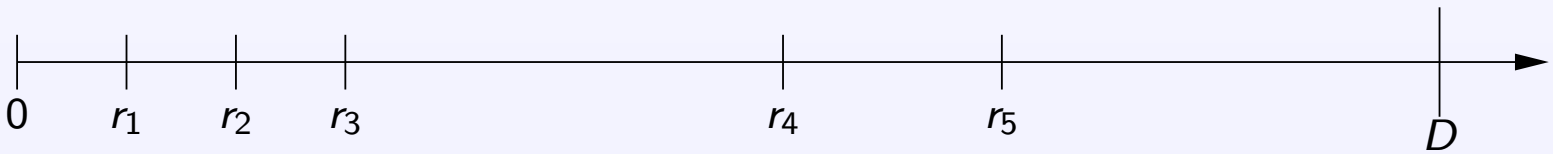
Polynomial cases

No release date	SPT ($O(NlogN)$)
Preemptive	SRPT ($O(NlogN)$)
Identical processing times	nondecreasing order of the release dates $O(NlogN)$

Polynomial cases

No release date	SPT ($O(NlogN)$)
Preemptive	SRPT ($O(NlogN)$)
Identical processing times	nondecreasing order of the release dates $O(NlogN)$

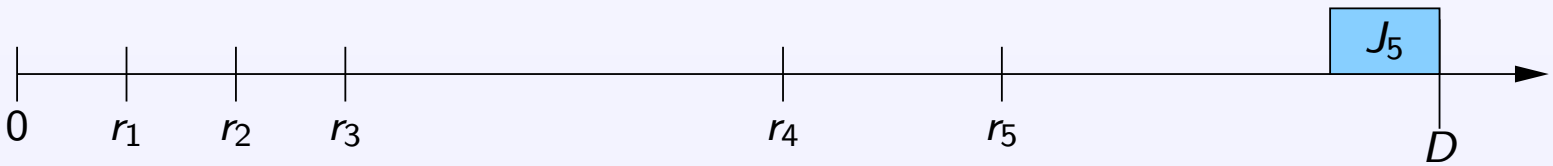
- If $\boldsymbol{K = 1}$, the problem can be solved in time $O(NlogN)$.
- Moore-Hodgson algorithm for $1||\sum U_i$: Moore, MS (98).



Polynomial cases

No release date	SPT ($O(NlogN)$)
Preemptive	SRPT ($O(NlogN)$)
Identical processing times	nondecreasing order of the release dates $O(NlogN)$

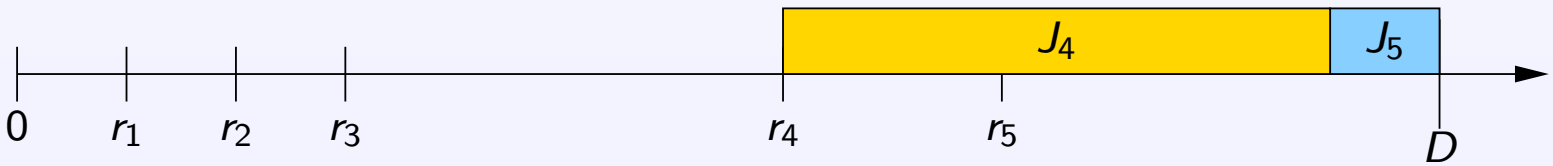
- If $\boldsymbol{K = 1}$, the problem can be solved in time $O(NlogN)$.
- Moore-Hodgson algorithm for $1||\sum U_i$: Moore, MS (98).



Polynomial cases

No release date	SPT ($O(NlogN)$)
Preemptive	SRPT ($O(NlogN)$)
Identical processing times	nondecreasing order of the release dates $O(NlogN)$

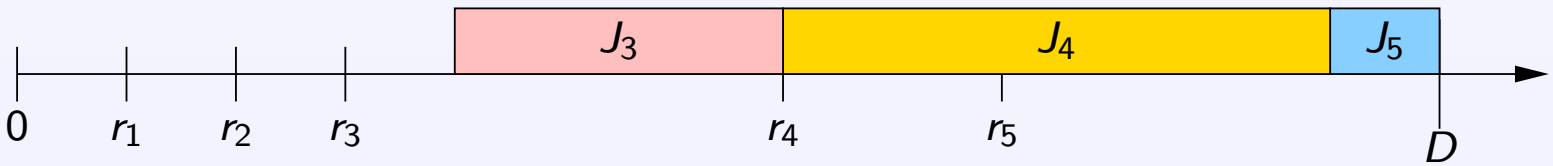
- If $\boldsymbol{K = 1}$, the problem can be solved in time $O(NlogN)$.
- Moore-Hodgson algorithm for $1||\sum U_i$: Moore, MS (98).



Polynomial cases

No release date	SPT ($O(NlogN)$)
Preemptive	SRPT ($O(NlogN)$)
Identical processing times	nondecreasing order of the release dates $O(NlogN)$

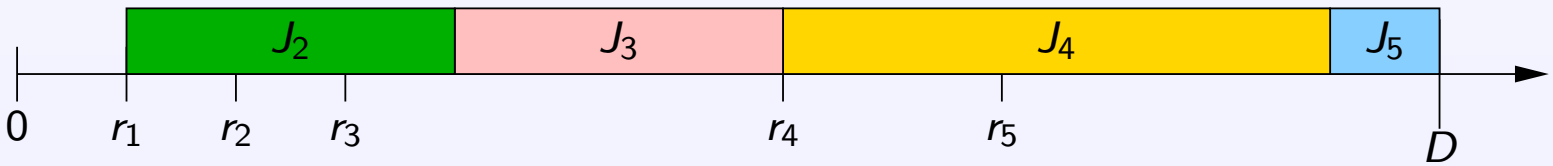
- If $\boldsymbol{K = 1}$, the problem can be solved in time $O(NlogN)$.
- Moore-Hodgson algorithm for $1||\sum U_i$: Moore, MS (98).



Polynomial cases

No release date	SPT ($O(NlogN)$)
Preemptive	SRPT ($O(NlogN)$)
Identical processing times	nondecreasing order of the release dates $O(NlogN)$

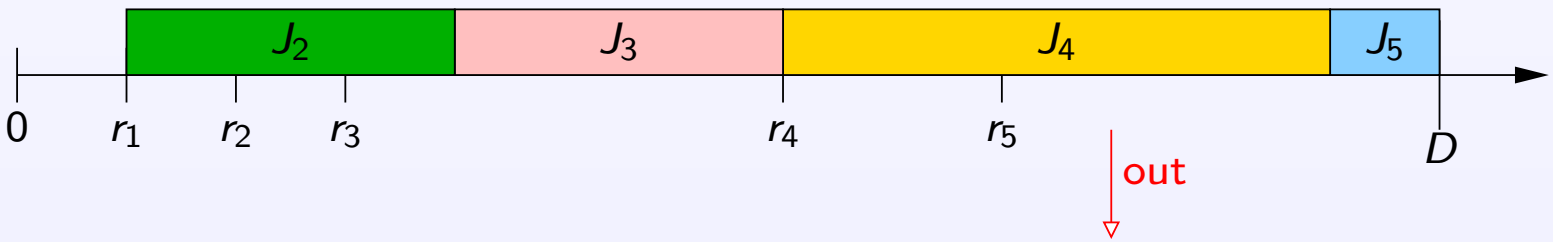
- If $\boldsymbol{K = 1}$, the problem can be solved in time $O(NlogN)$.
- Moore-Hodgson algorithm for $1||\sum U_i$: Moore, MS (98).



Polynomial cases

No release date	SPT ($O(NlogN)$)
Preemptive	SRPT ($O(NlogN)$)
Identical processing times	nondecreasing order of the release dates $O(NlogN)$

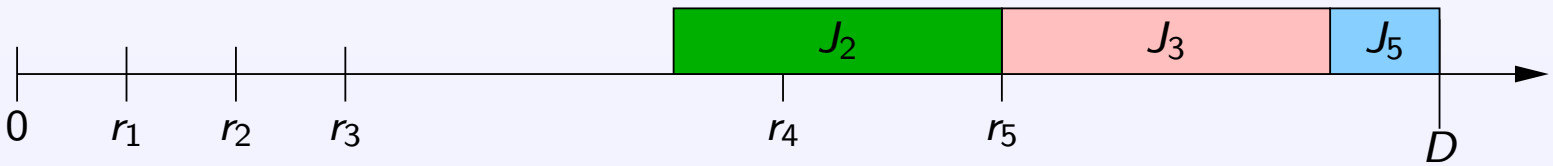
- If $\boldsymbol{K = 1}$, the problem can be solved in time $O(NlogN)$.
- Moore-Hodgson algorithm for $1||\sum U_i$: Moore, MS (98).



Polynomial cases

No release date	SPT ($O(NlogN)$)
Preemptive	SRPT ($O(NlogN)$)
Identical processing times	nondecreasing order of the release dates $O(NlogN)$

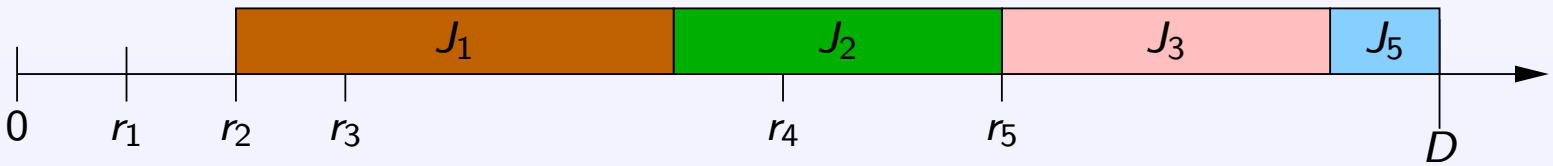
- If $\boldsymbol{K = 1}$, the problem can be solved in time $O(NlogN)$.
- Moore-Hodgson algorithm for $1||\sum U_i$: Moore, MS (98).



Polynomial cases

No release date	SPT ($O(NlogN)$)
Preemptive	SRPT ($O(NlogN)$)
Identical processing times	nondecreasing order of the release dates $O(NlogN)$

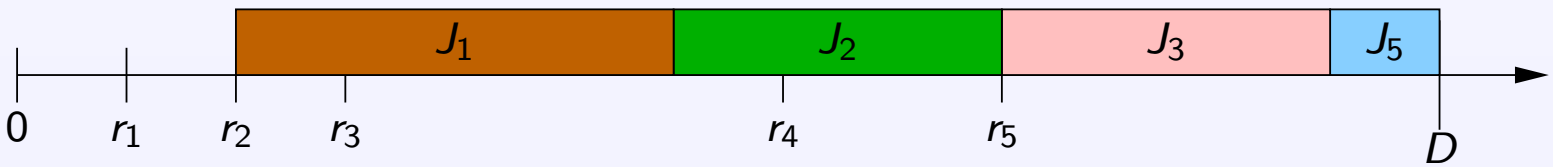
- If $\boldsymbol{K = 1}$, the problem can be solved in time $O(NlogN)$.
- Moore-Hodgson algorithm for $1||\sum U_i$: Moore, MS (98).



Polynomial cases

No release date	SPT ($O(NlogN)$)
Preemptive	SRPT ($O(NlogN)$)
Identical processing times	nondecreasing order of the release dates $O(NlogN)$

- If $\boldsymbol{K = 1}$, the problem can be solved in time $O(NlogN)$.
- Moore-Hodgson algorithm for $1||\sum U_i$: Moore, MS (98).



- $1|r_i|V$ equivalent to $1|r_i, d_i = d|\sum U_i$.

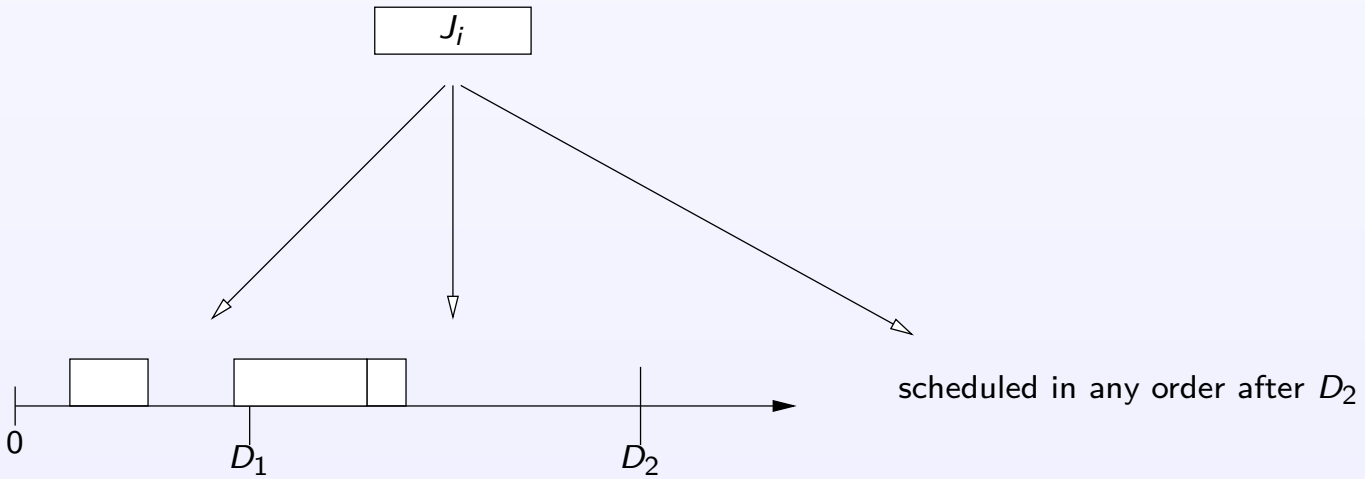
Dominance rules

There exists an optimal solution such that :

1. there is no idle time between any pair of consecutive jobs scheduled between D_1 and D_2,
2. the jobs scheduled between two consecutive delivery dates (with $D_0 = 0$) are ordered following their nondecreasing release dates.

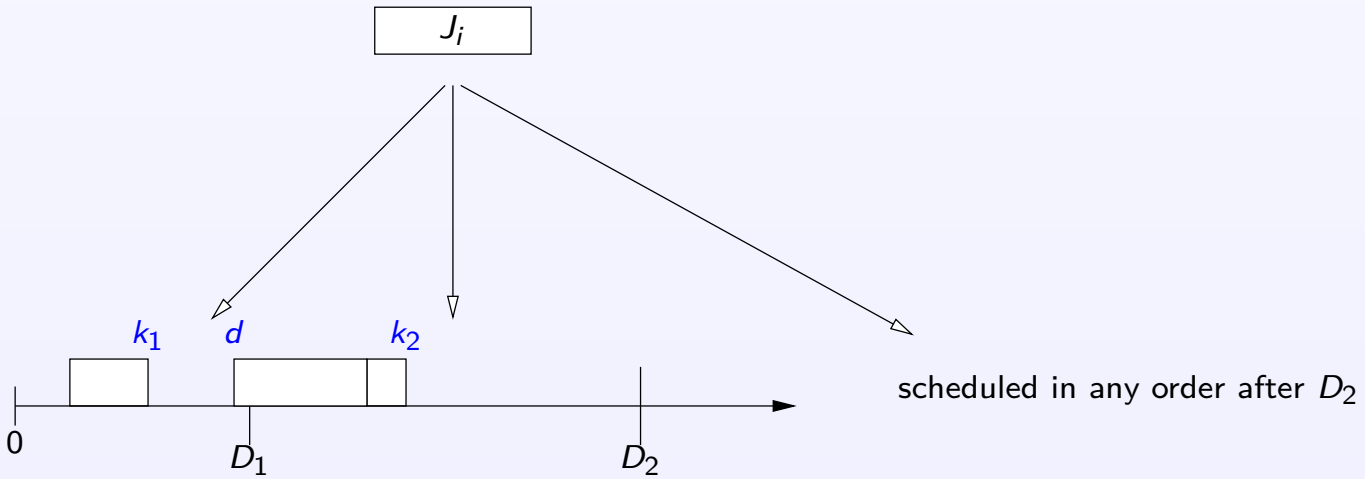
Dynamic programming algorithm

- Dynamic programming algorithm for $1|r_i|V_1 + V_2$.
- Rationale of the algorithm :
  - Jobs are ordered following their nondecreasing release dates : $J_1, \ldots, J_N$
  - N steps
  - Step i :



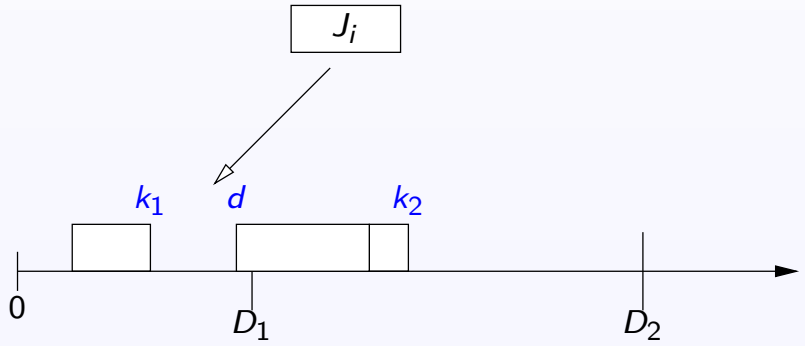
Dynamic programming algorithm

- Dynamic programming algorithm for $1|r_i|V_1 + V_2$.
- Rationale of the algorithm :
  - Jobs are ordered following their nondecreasing release dates : $J_1, \ldots, J_N$
  - N steps
  - Step i :



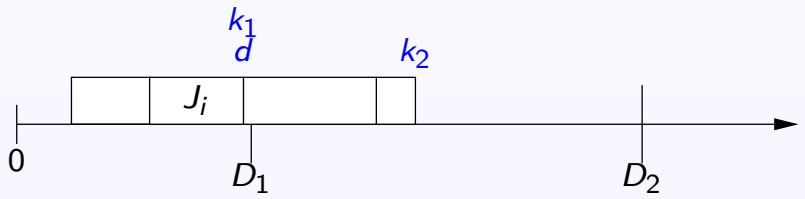
Dynamic programming algorithm

Case 1 :



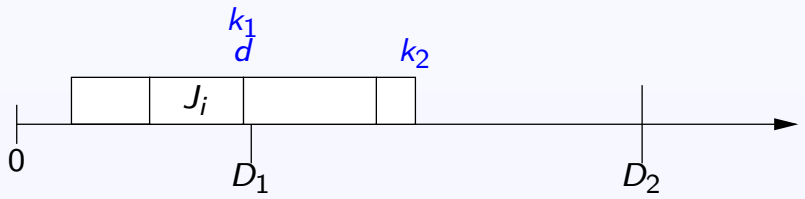
Dynamic programming algorithm

Case 1 :

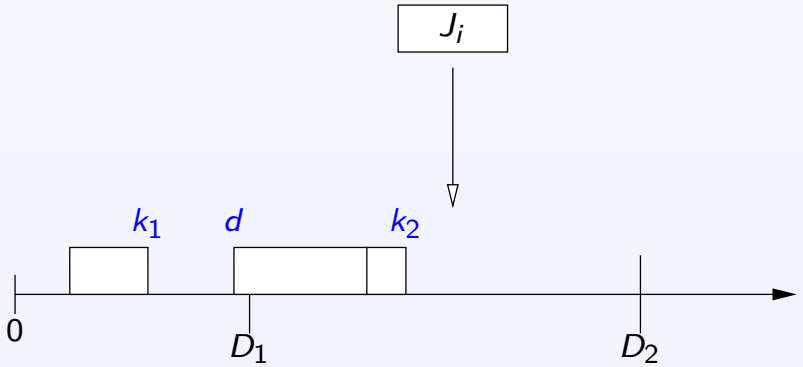


Dynamic programming algorithm

Case 1 :

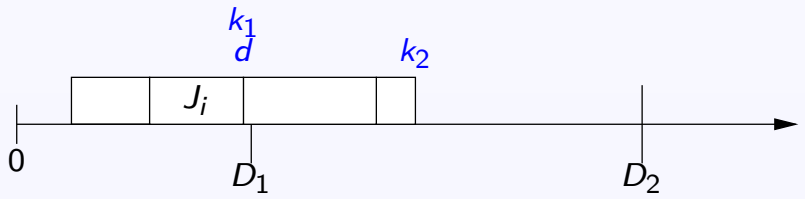


Case 2 :

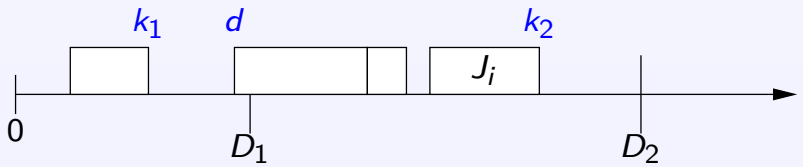


Dynamic programming algorithm

Case 1 :

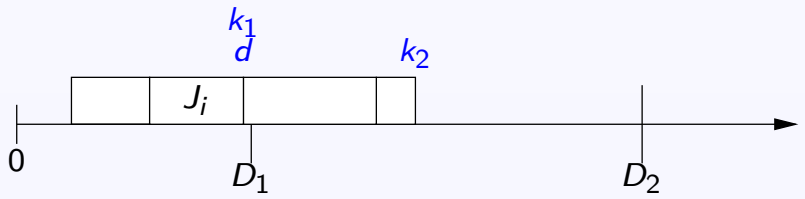


Case 2 :

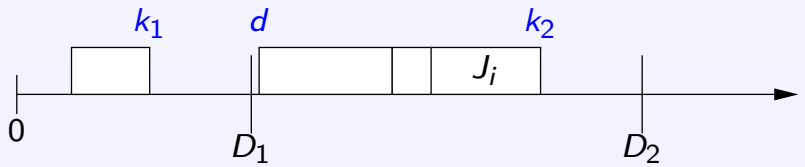


Dynamic programming algorithm

Case 1 :

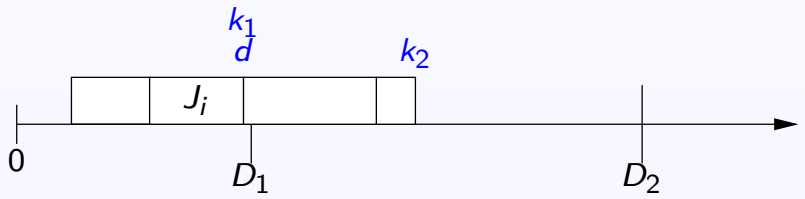


Case 2 :

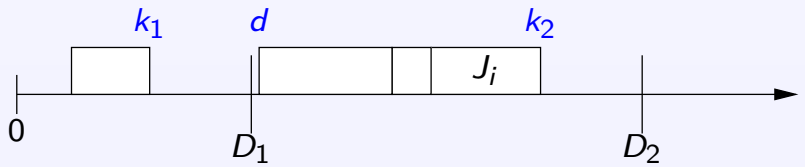


Dynamic programming algorithm

Case 1 :



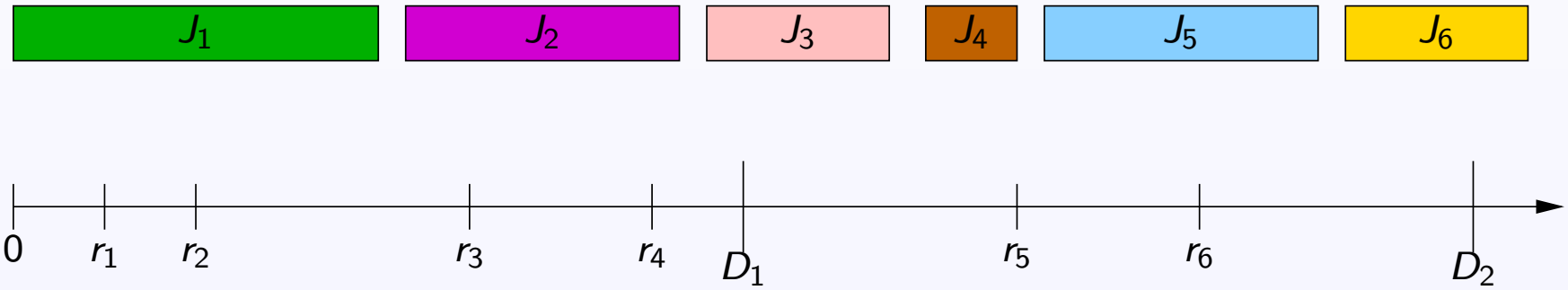
Case 2 :



Complexity : $O(N(D_1(D_2)^2) + N \log N)$

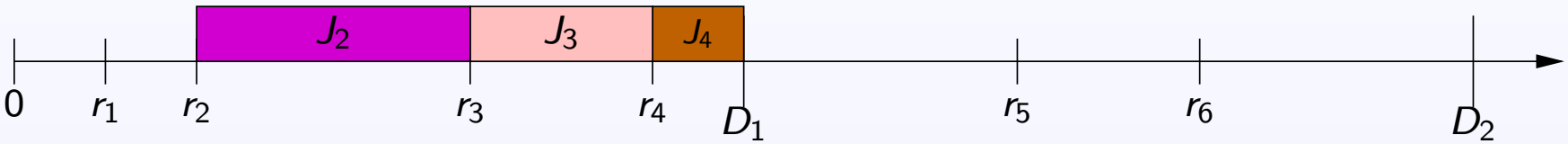
Additional dominance rule

Given an instance of $1|r_i|V_1 + V_2$:



Additional dominance rule

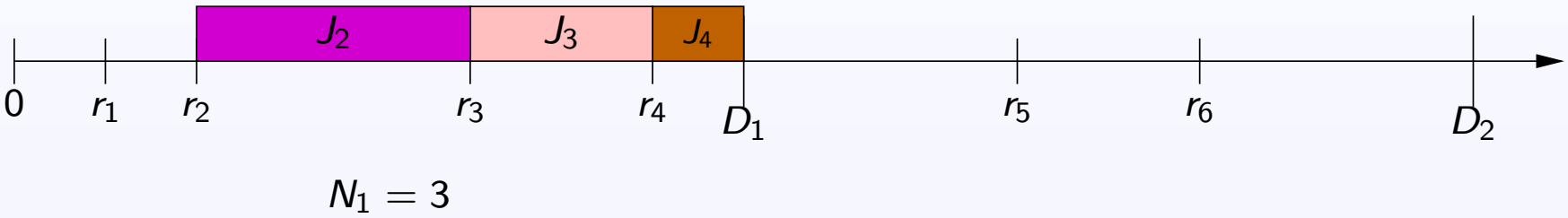
Given an instance of $1|r_i|V_1 + V_2$:



$N_1 = 3$

Additional dominance rule

Given an instance of $1|r_i|V_1 + V_2$:

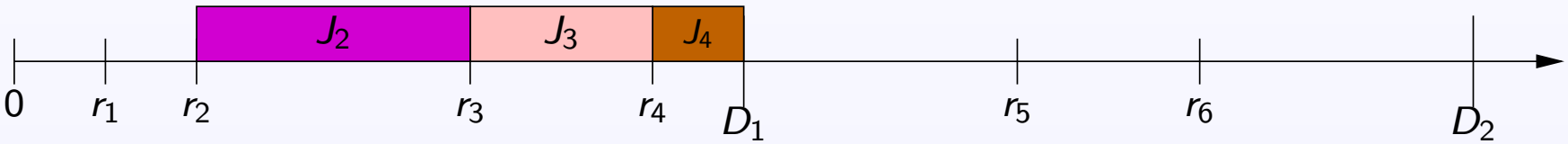


$N_1 = 3$

Dominance rule: there exists an optimal solution in which N_1 jobs complete before D_1.

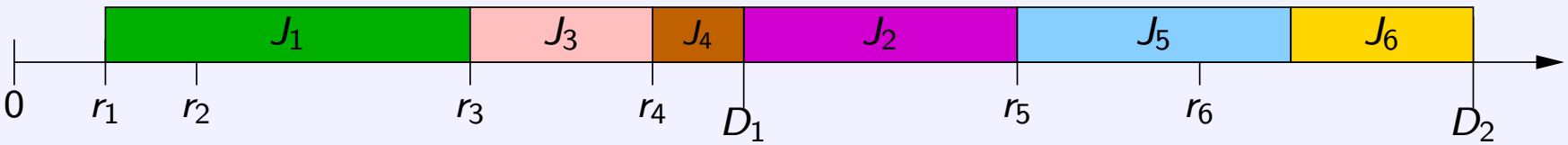
Additional dominance rule

Given an instance of $1|r_i|V_1 + V_2$:



$N_1 = 3$

Dominance rule: there exists an optimal solution in which N_1 jobs complete before D_1.



Preliminary results

Instance generation scheme:

- Processing times $p_i \in [10, 100[$
- Delivery dates: $\{T, 2T\}$, with $T = \alpha \sum p_i/2$ ($\alpha \in \{0.8, 1, 1.2\}$)
- Release dates in $[0, r_{amp}\,T]$ or $[T, T + r_{amp}\,T]$ ($r_{amp} \in \{0.1, 0.3, 0.5\}$)

Preliminary results

Instance generation scheme:

- Processing times $p_i \in [10, 100[$
- Delivery dates: $\{T, 2T\}$, with $T = \alpha \sum p_i/2$ ($\alpha \in \{0.8, 1, 1.2\}$)
- Release dates in $[0, r_{amp}\, T]$ or $[T, T + r_{amp}\, T]$ ($r_{amp} \in \{0.1, 0.3, 0.5\}$)

Simple dynamic programming:

Mean CPU time on 45 30-jobs instances (time limit: 15 min CPU):

344.7 (41/45)

Preliminary results

Instance generation scheme:

- Processing times $p_i \in [10, 100[$
- Delivery dates: $\{T, 2T\}$, with $T = \alpha \sum p_i/2$ ($\alpha \in \{0.8, 1, 1.2\}$)
- Release dates in $[0, r_{amp}\,T]$ or $[T, T + r_{amp}\,T]$ ($r_{amp} \in \{0.1, 0.3, 0.5\}$)

Simple dynamic programming:

Mean CPU time on 45 30-jobs instances (time limit: 15 min CPU):

344.7 (41/45)

Dynamic programming + additional dominance rule:

Mean CPU time on 45 N-jobs instances (time limit: 30 min CPU):

N	30	40	50	60	70	80
CPU time (s)	9.3	56.7	274.8	668.4 (44) (693.5)	797.8 (28) (1176.4)	1186.6 (9) (1677.3)

3.33 GHz Intel Core2-Duo processor, 8 GB RAM, running Debian wheezy/sid

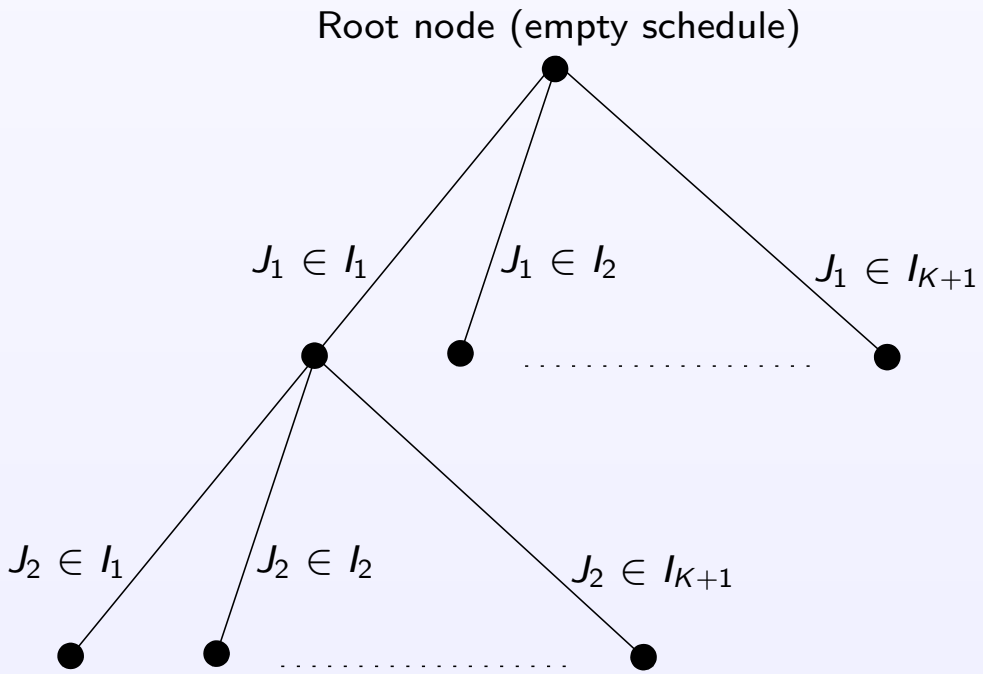
Dominance rules

There exists an optimal solution such that :

1. there is no idle time between any pair of consecutive jobs scheduled between D_k and D_{k+1}, $k = 0, \ldots, K-1$
2. the jobs scheduled between two consecutive delivery dates (with $D_0 = 0$) are ordered following their nondecreasing release dates.

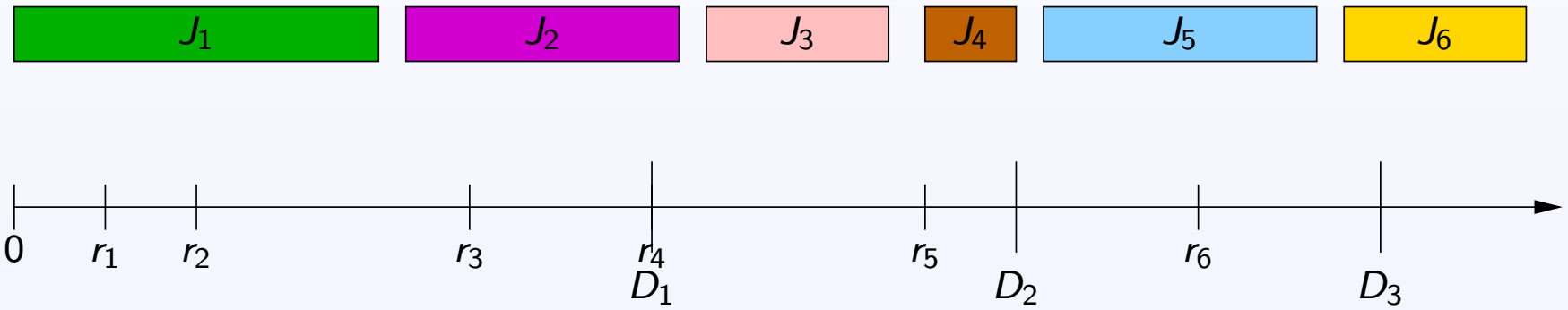
Branching rule

- Jobs numbered by nondecreasing release dates : $J_1, \ldots, J_N$
- $I_k =]D_{k-1}, D_k]$, $k = 1, \ldots, K$
- $I_{K+1} =]D_K, \max_{i=1,\ldots,N} r_i + \sum_{i=1}^{N} p_i]$



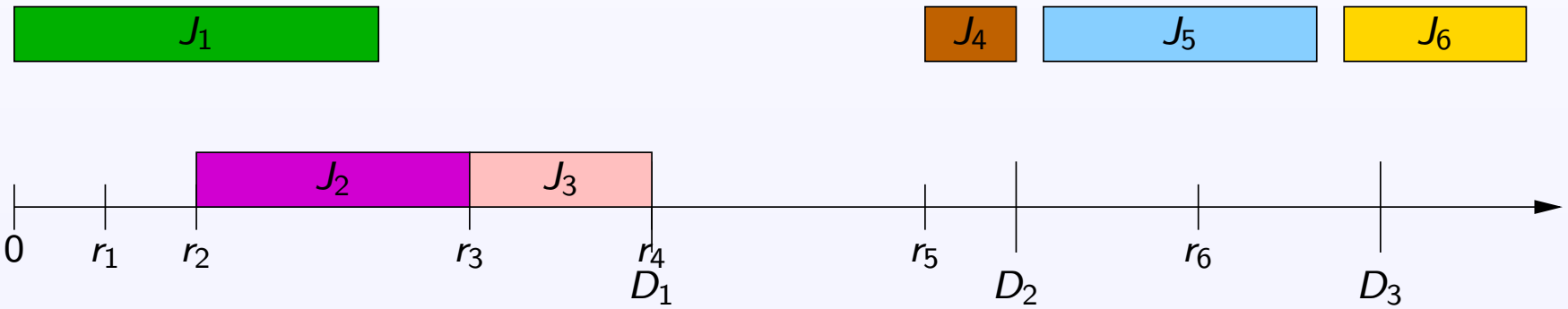
An initial lower bound

- Algorithm for $1|r_i|V$ applied on each I_k
- on not yet scheduled jobs



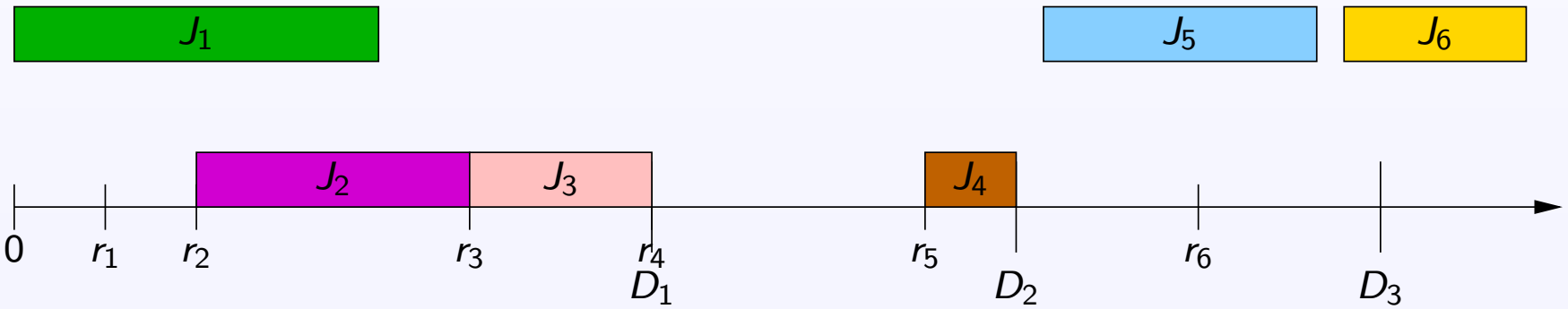
An initial lower bound

- Algorithm for $1|r_i|V$ applied on each I_k
- on not yet scheduled jobs



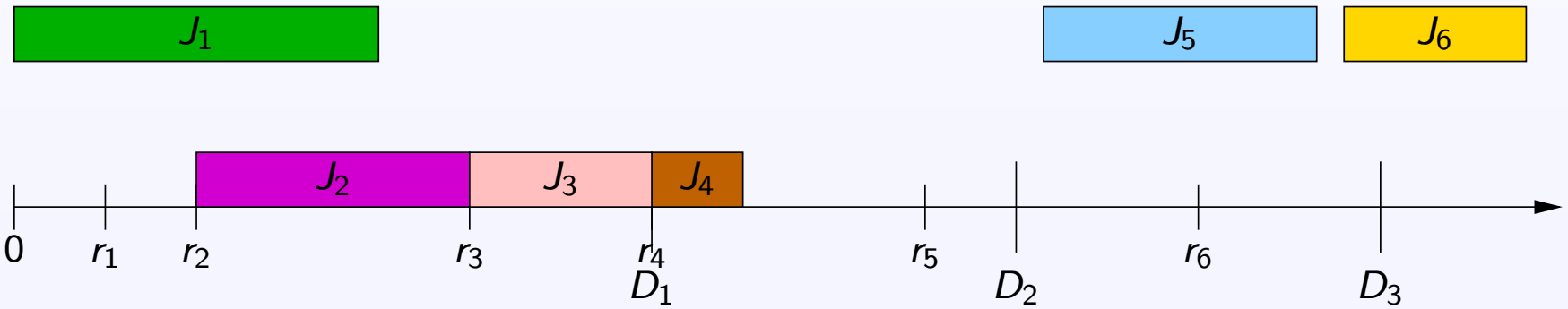
An initial lower bound

- Algorithm for $1|r_i|V$ applied on each I_k
- on not yet scheduled jobs



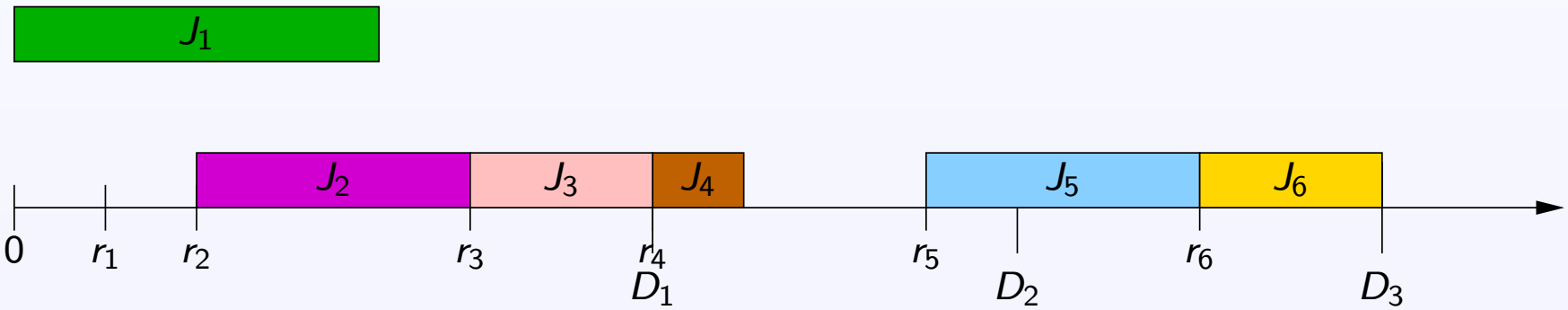
An initial lower bound

- Algorithm for $1|r_i|V$ applied on each I_k
- on not yet scheduled jobs



An initial lower bound

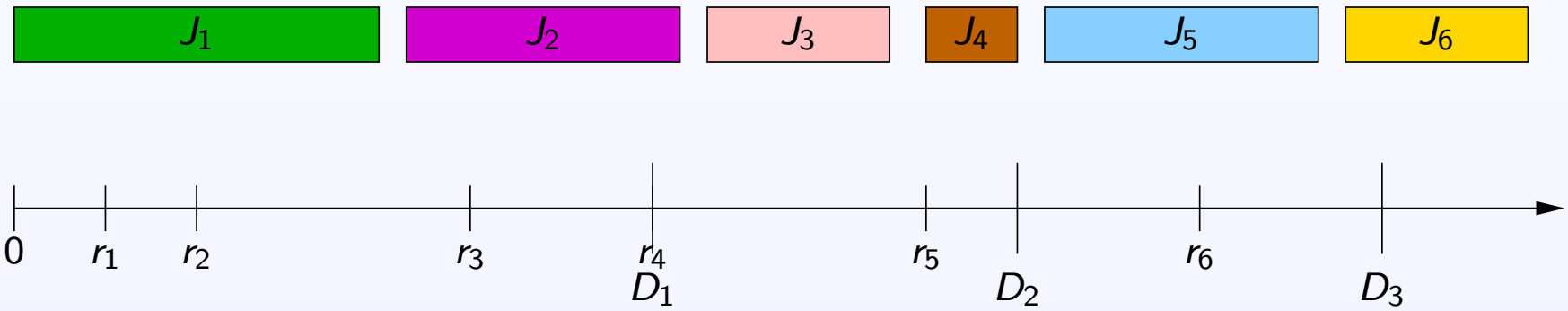
- Algorithm for $1|r_i|V$ applied on each I_k
- on not yet scheduled jobs



- 2 jobs in $I_1 \rightarrow 2 \times 3 = 6$
- 1 job in $I_2 \rightarrow 1 \times 2 = 2$
- 2 jobs in $I_3 \rightarrow 2 \times 1 = 2$
- Payoff : 10

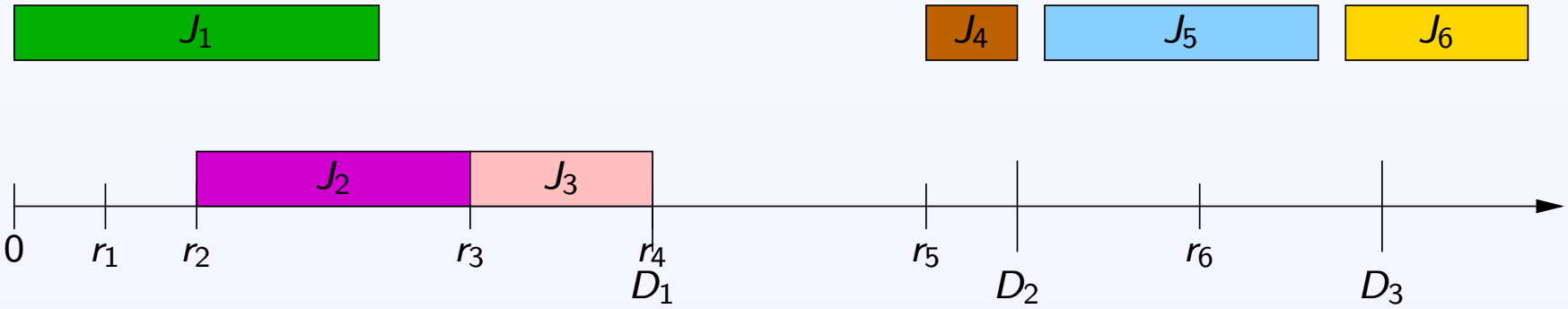
An initial upper bound

- Algorithm for $1|r_i|V$ applied on each interval $[0, D_k]$, $k = 1, \ldots, K$
- always on the initial set of jobs



An initial upper bound

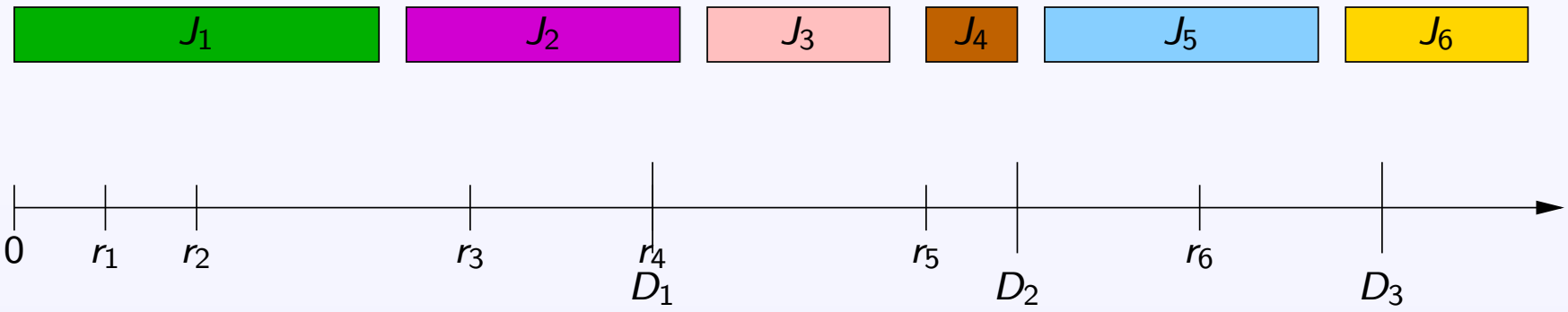
- Algorithm for $1|r_i|V$ applied on each interval $[0, D_k]$, $k = 1, \ldots, K$
- always on the initial set of jobs



Maximum number of jobs in $[0, D_1]$: $N_1 = 2$

An initial upper bound

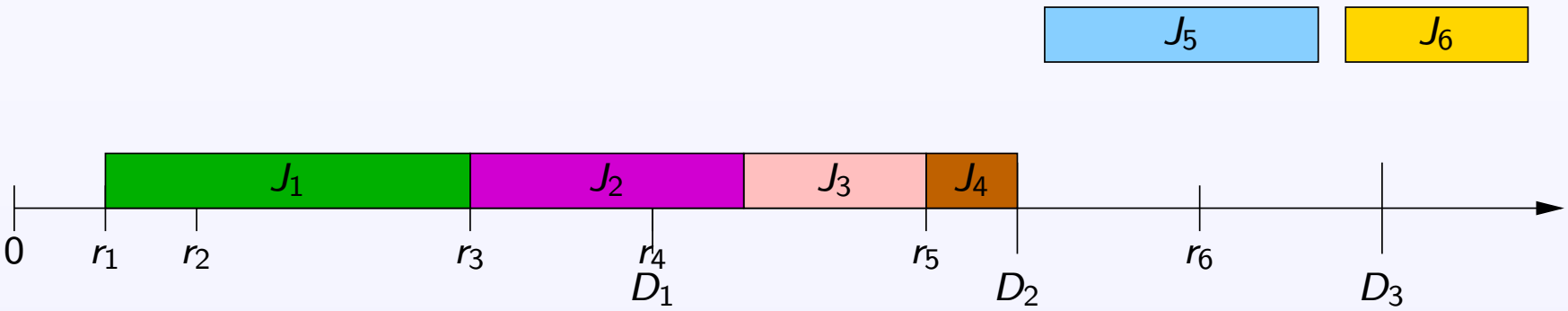
- Algorithm for $1|r_i|V$ applied on each interval $[0, D_k]$, $k = 1, \ldots, K$
- always on the initial set of jobs



Maximum number of jobs in $[0, D_1]$: $N_1 = 2$

An initial upper bound

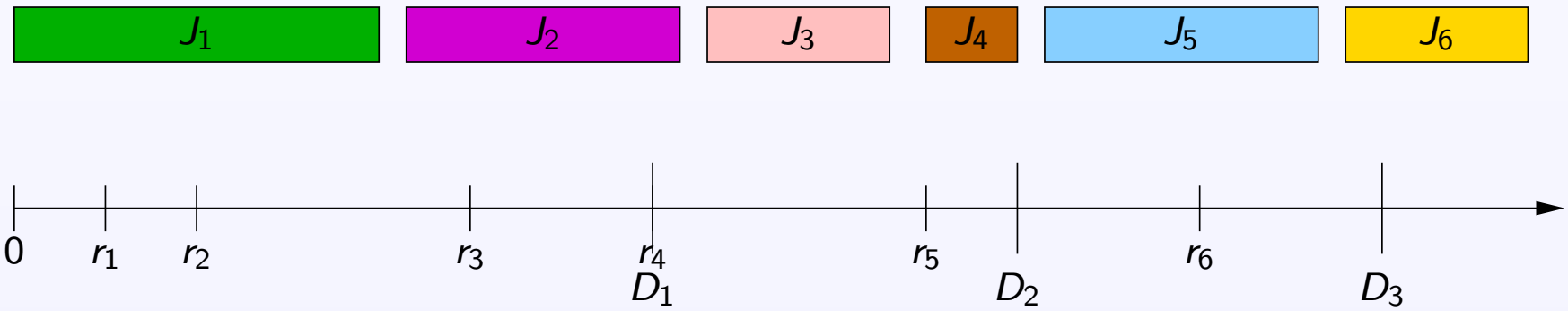
- Algorithm for $1|r_i|V$ applied on each interval $[0, D_k]$, $k = 1, \ldots, K$
- always on the initial set of jobs



Maximum number of jobs in $[0, D_1]$: $N_1 = 2$
Maximum number of jobs in $[0, D_2]$: $N_2 = 4$

An initial upper bound

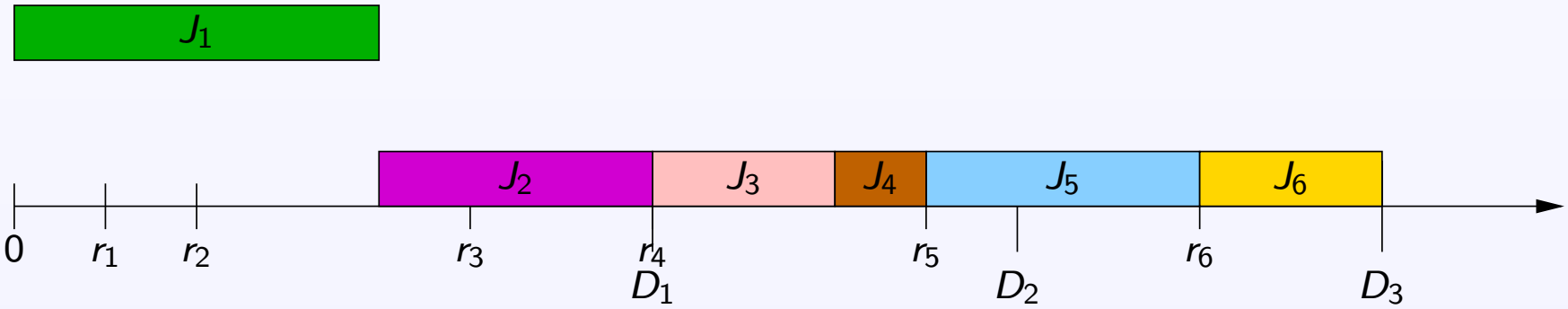
- Algorithm for $1|r_i|V$ applied on each interval $[0, D_k]$, $k = 1, \ldots, K$
- always on the initial set of jobs



Maximum number of jobs in $[0, D_1]$: $N_1 = 2$
Maximum number of jobs in $[0, D_2]$: $N_2 = 4$

An initial upper bound

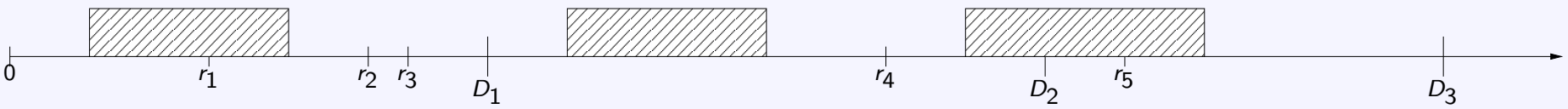
- Algorithm for $1|r_i|V$ applied on each interval $[0, D_k]$, $k = 1, \ldots, K$
- always on the initial set of jobs



Maximum number of jobs in $[0, D_1]$: $N_1 = 2$
Maximum number of jobs in $[0, D_2]$: $N_2 = 4$
Maximum number of jobs in $[0, D_3]$: $N_3 = 5$
Upper bound : N_1 jobs in I_1, $N_2 - N_1$ jobs in I_2, $N_3 - N_2$ jobs in I_3
Payoff : $2 \times 3 + 2 \times 2 + 1 \times 1 = 11$

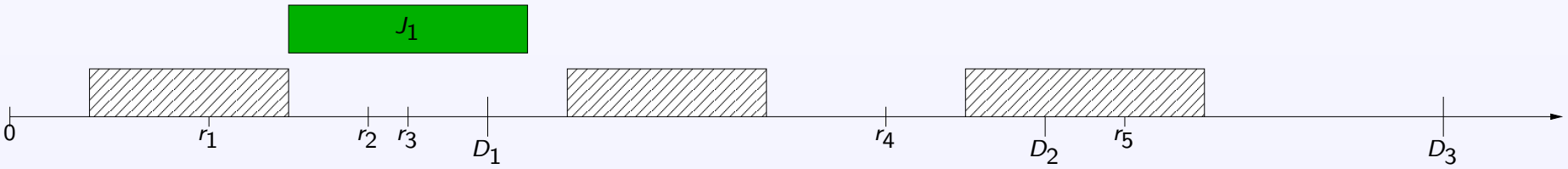
An upper bound on a partial schedule

Completion with SRPT rule :



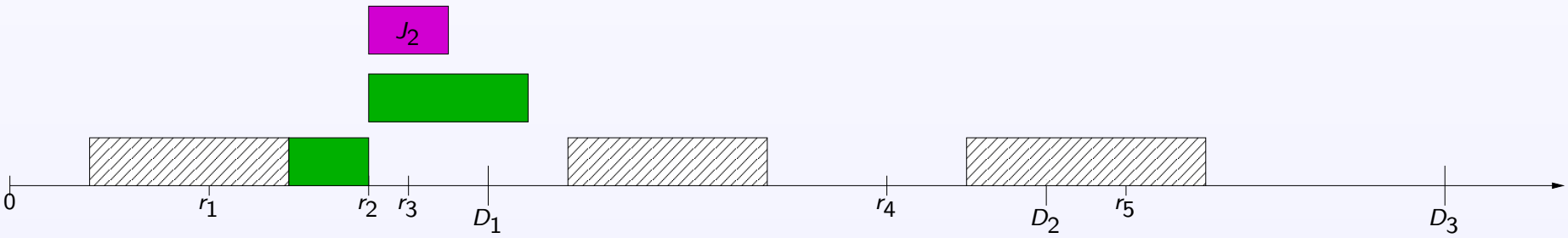
An upper bound on a partial schedule

Completion with SRPT rule :



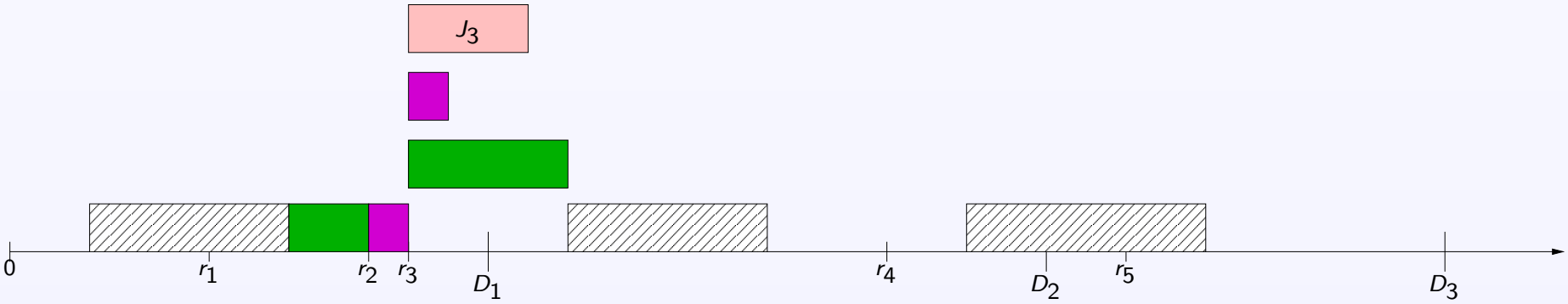
An upper bound on a partial schedule

Completion with SRPT rule :



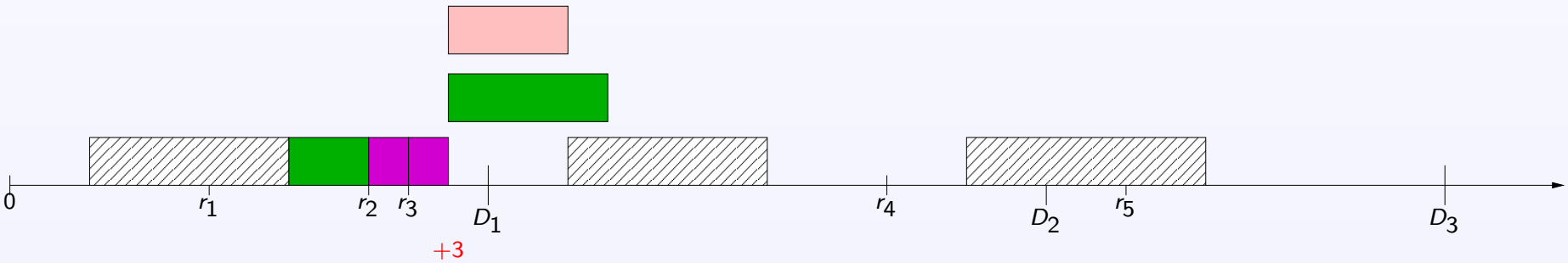
An upper bound on a partial schedule

Completion with SRPT rule :



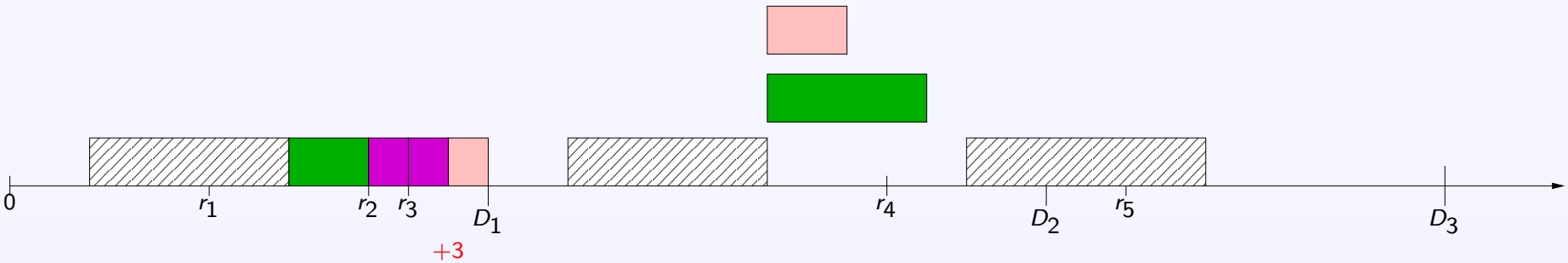
An upper bound on a partial schedule

Completion with SRPT rule :



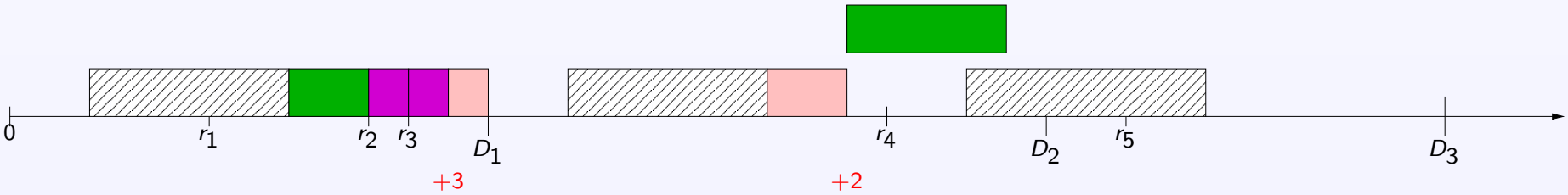
An upper bound on a partial schedule

Completion with SRPT rule :



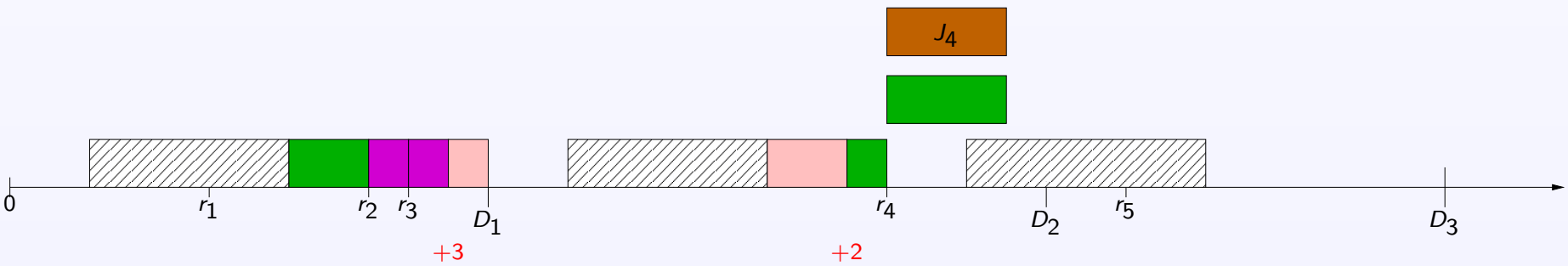
An upper bound on a partial schedule

Completion with SRPT rule :



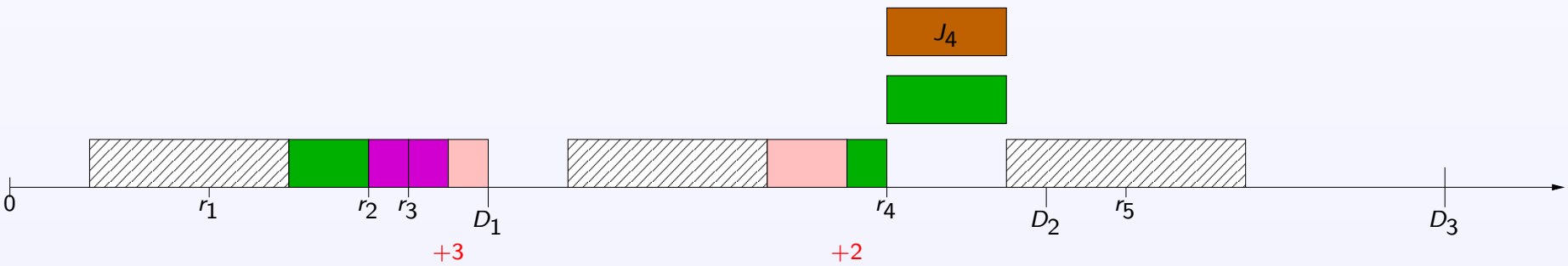
An upper bound on a partial schedule

Completion with SRPT rule :



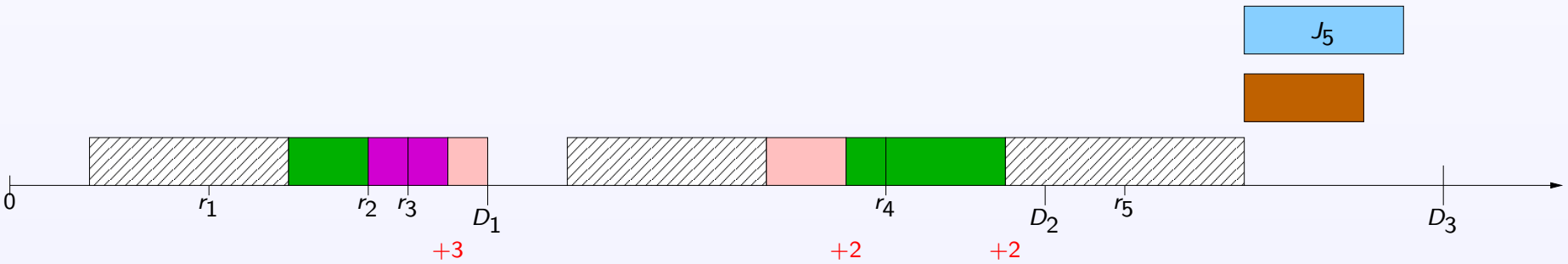
An upper bound on a partial schedule

Completion with SRPT rule :



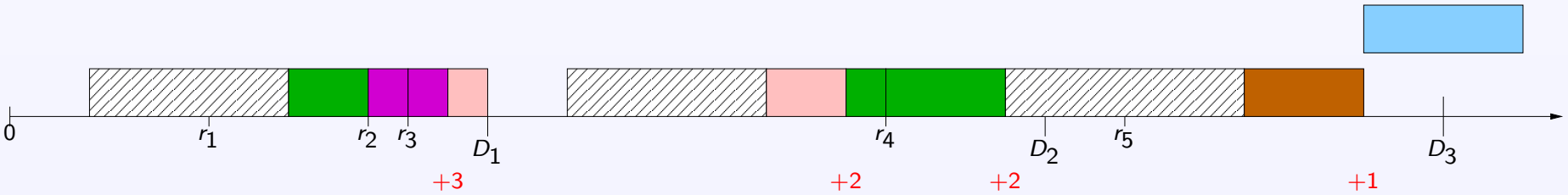
An upper bound on a partial schedule

Completion with SRPT rule :



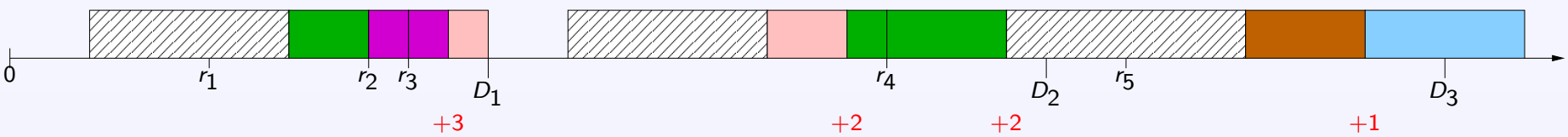
An upper bound on a partial schedule

Completion with SRPT rule :



An upper bound on a partial schedule

Completion with SRPT rule :



Preliminary results

Instance generation scheme:

- Processing times $p_i \in [10, 100[$
- Delivery dates: $\{T, 2T, \ldots, KT\}$, with $T = \lfloor \alpha \sum p_i / K \rfloor$ ($\alpha \in \{0.8, 1, 1.2\}$)
- Release dates in $[(k-1)T, (k-1)T + r_{amp}T]$, $k = 1, \ldots, K$ ($r_{amp} \in \{0.1, 0.3, 0.5, 0.7\}$)

Preliminary results

Instance generation scheme:

- Processing times $p_i \in [10, 100[$
- Delivery dates: $\{T, 2T, \ldots, KT\}$, with $T = \lfloor \alpha \sum p_i / K \rfloor$ ($\alpha \in \{0.8, 1, 1.2\}$)
- Release dates in $[(k-1)T, (k-1)T + r_{amp}T]$, $k = 1, \ldots, K$ ($r_{amp} \in \{0.1, 0.3, 0.5, 0.7\}$)

Number of instances solved in less than 3600 s. CPU :

$K \backslash N$	100	200	300	500
2 (200 inst.)	196 (194)	193 (193)	198 (198)	199 (199)
3 (60 inst.)	58 (57)	58 (58)	59 (59)	60 (59)
4 (60 inst.)	55 (54)	54 (54)	54 (54)	56 (56)

Mean CPU time for $N = 100$:

K	2 (2 inst.)	3 (1 inst.)	4 (1 inst.)
Time	1032.00	1049.56	5.87

3.33 GHz Intel Core2-Duo processor, 8 GB RAM, running Debian wheezy/sid

Travaux en cours

- Dans le Branch and Bound, calcul de la borne supérieure d'une solution partielle avec un algorithme analogue à celui calculant une borne supérieure initiale
- Comparaison avec les résultats de Detienne et al. (2011)
- Recherche d'un algorithme approché avec garantie de performance pour $1|r_i|V_1 + V_2$


B. Detienne, S. Dauzères-Pérès, and C. Yugma.
Scheduling jobs on parallel machine to minimize a regular step total cost function.
Journal of Scheduling, 2011.


N. G. Hall, M. Lesaoana, and C. N. Potts.
Scheduling with fixed delivery dates.
Operations Research, 49(1), 2001.


N. G. Hall, S. P. Sethi, and C. Sriskandarajah.
On the complexity of generalized due date scheduling problems.
European Journal of Operational Research, 51(1):100–109, March 1991.


A. Janiak and T. Krysiak.
Single processor scheduling with job values depending on their completion times.
Journal of Scheduling, 10(2):129–138, April 2007.


J. M. Moore.
An n Job, One Machine Sequencing Algorithm for Minimizing the Number of Late Jobs.
Management Science, 15(1):102–109, 1968.