Some ideas and open problems in real-time stochastic scheduling

Liliana CUCU, TRIO team, Nancy, France

Real-time systems

- ☑ Reactive systems
- ☑ Correct reaction
- ☑ Temporal constraints

Real-time systems (2)

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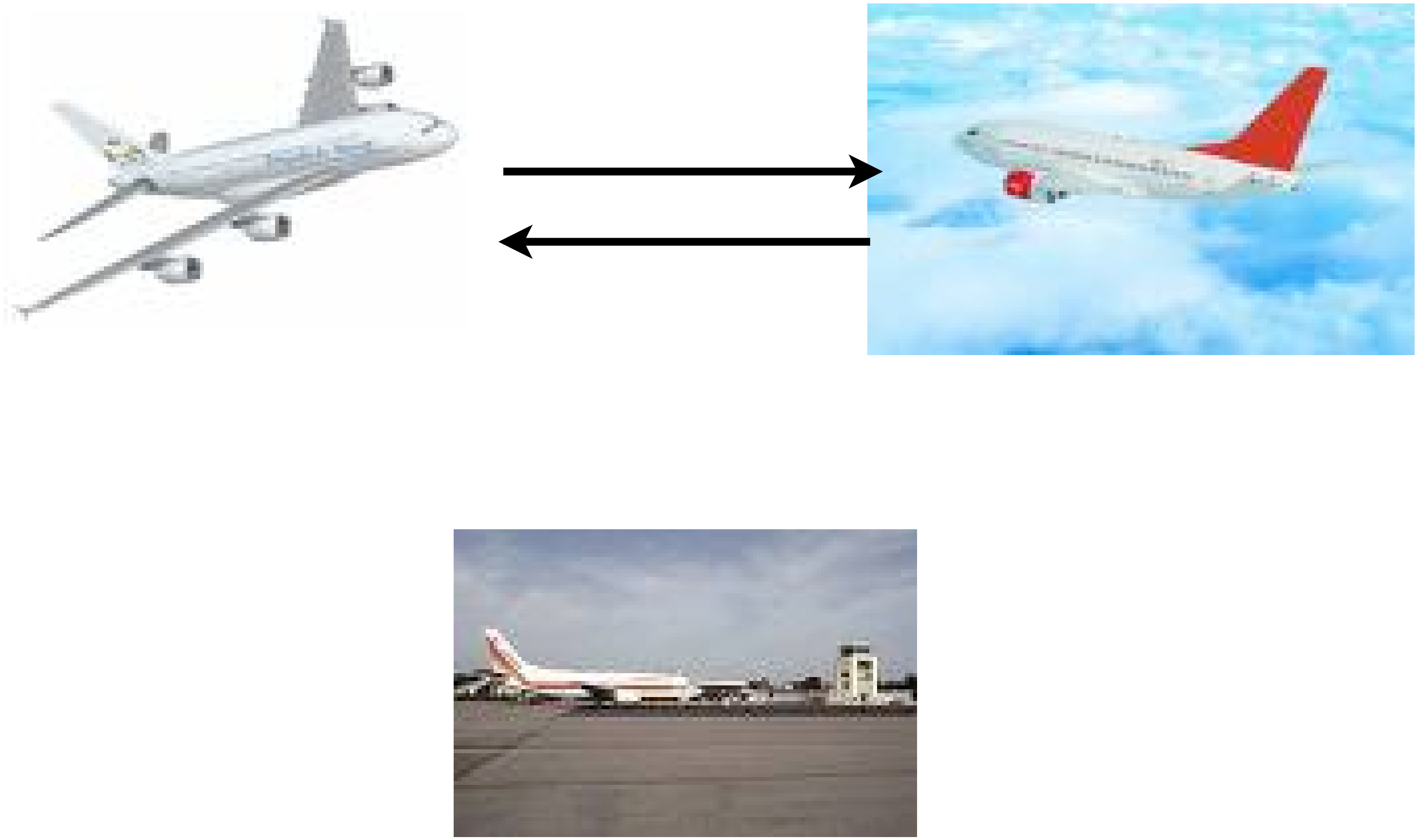

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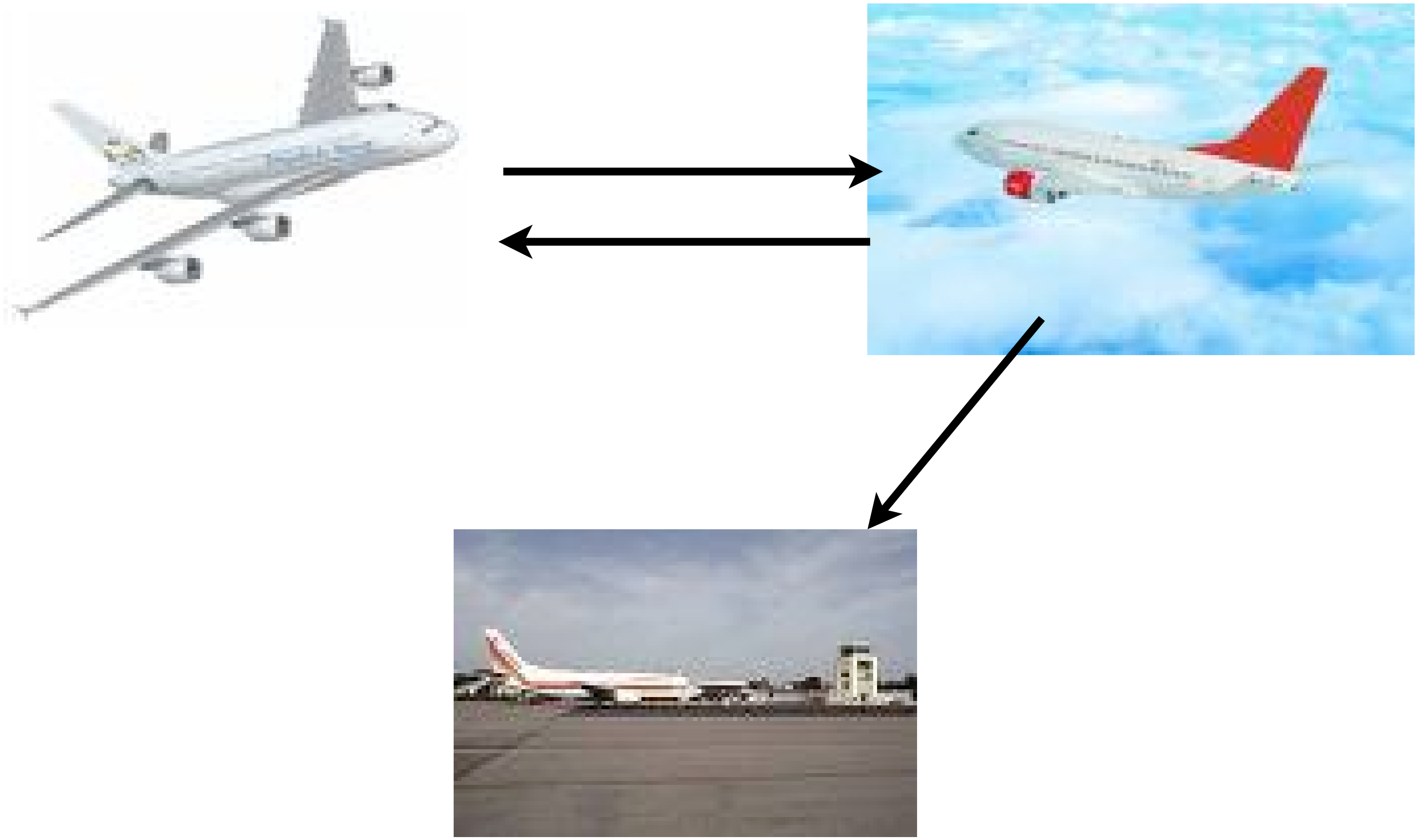

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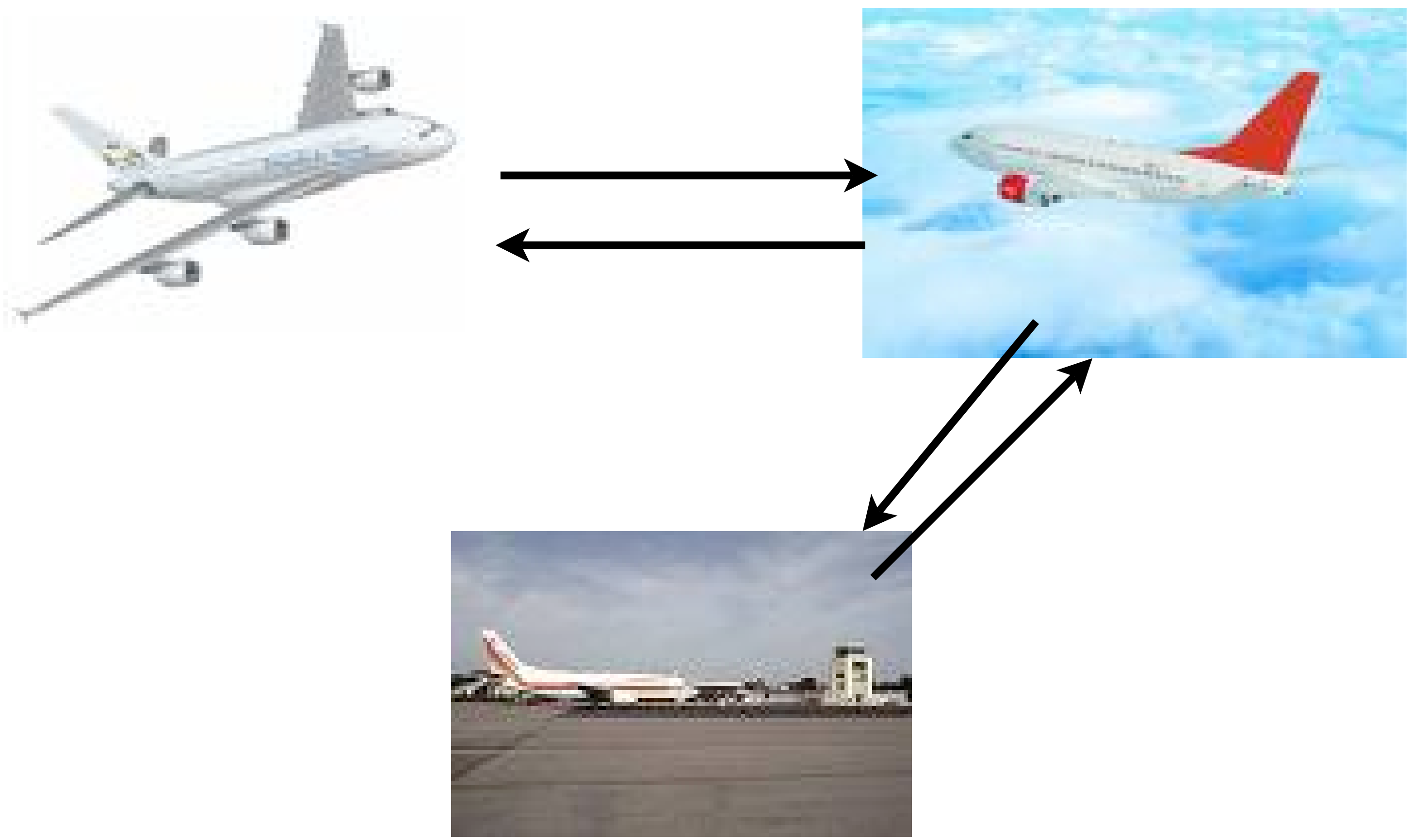

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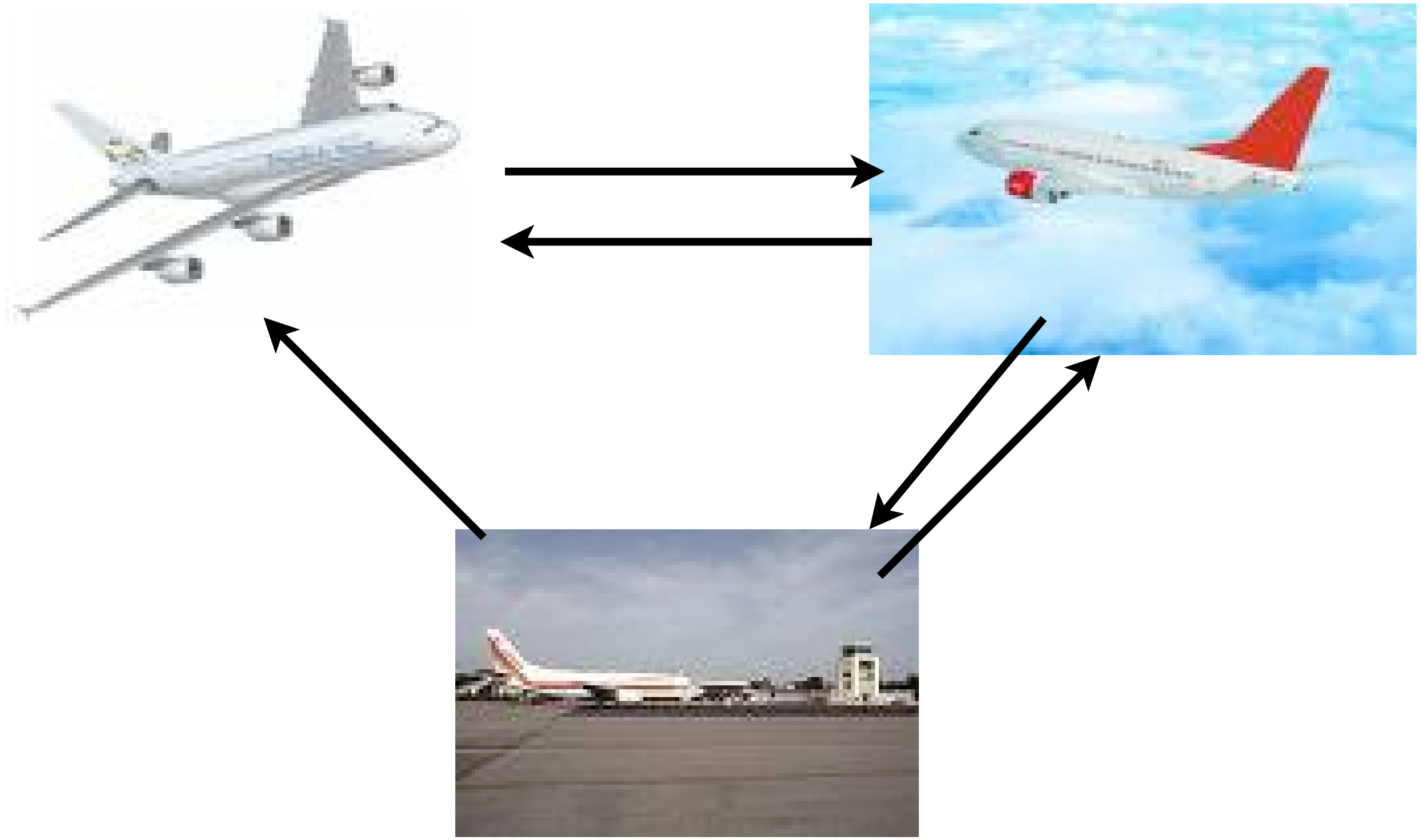

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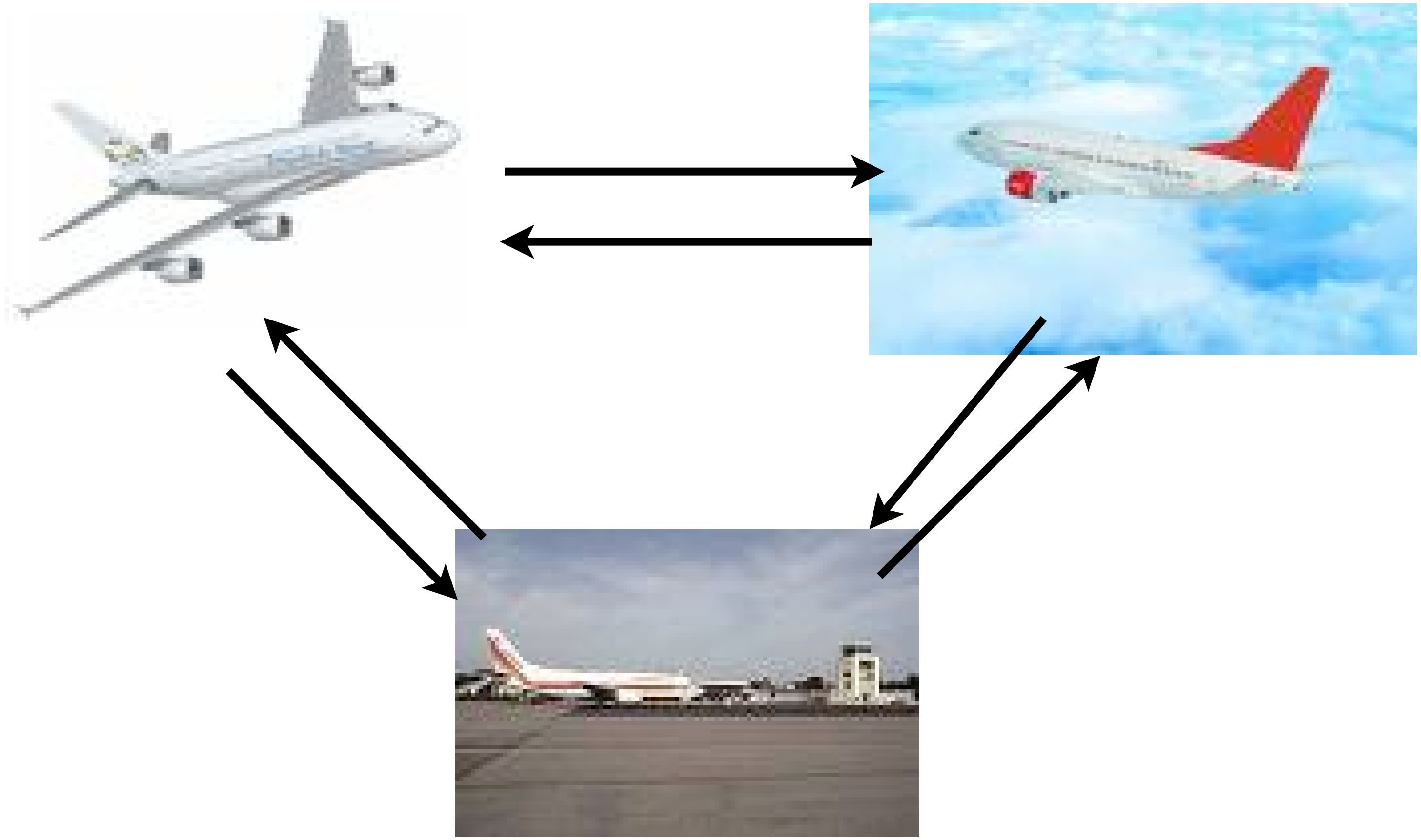
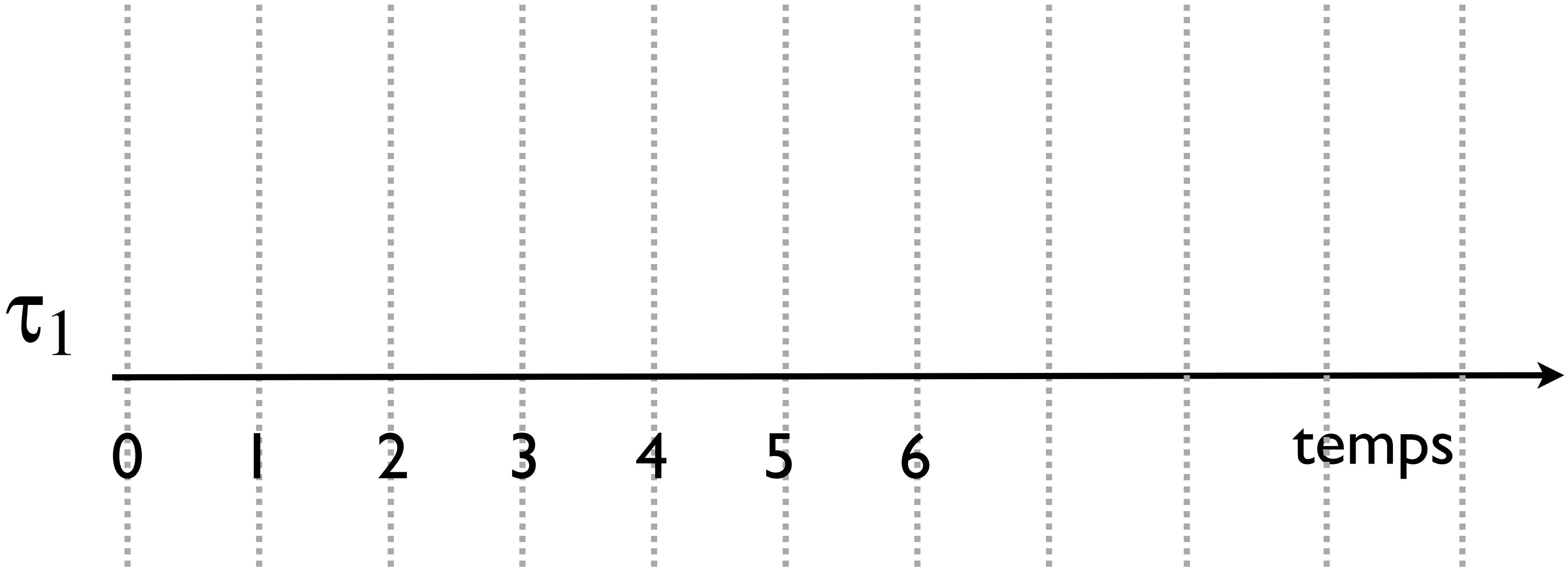


Real-time model: $\tau_i = (O_i, C_i, T_i, D_i)$

$$\tau_1 = (O_1, C_1, T_1, D_1) = (1, 2, 5, 4)$$

↑ release times

↓ deadlines



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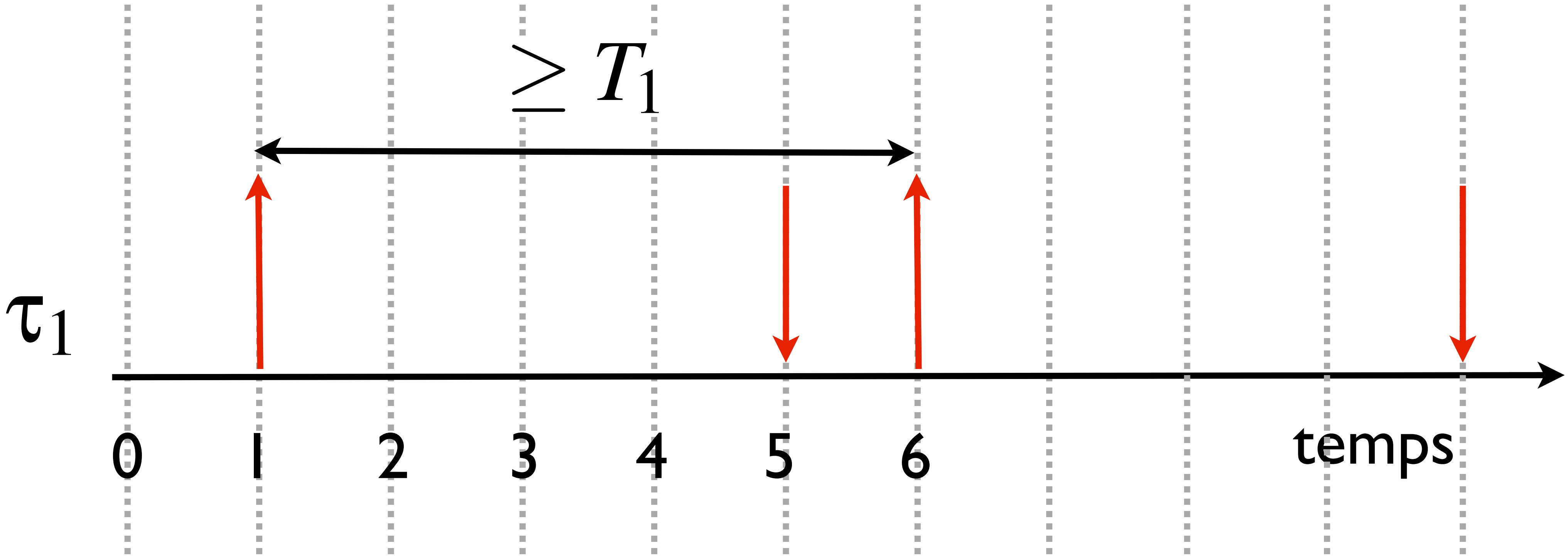

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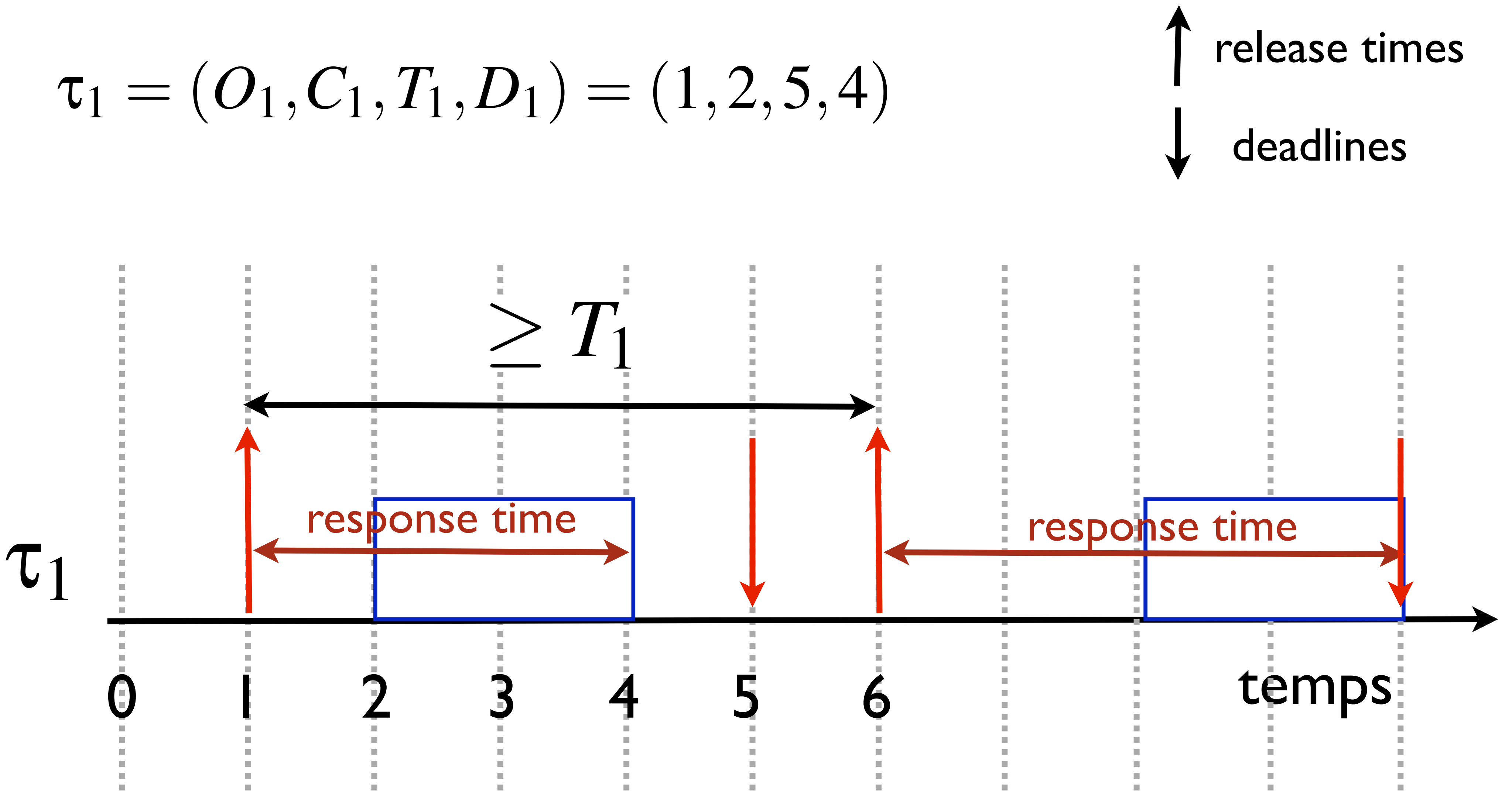

Why stochastic?

- Soft real-time constraints
- Uncertainness
- Worst-case behavior is a rare event

Where is the “stochastic touch”?

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Extracting quantitative information, i.e., obtaining distribution functions

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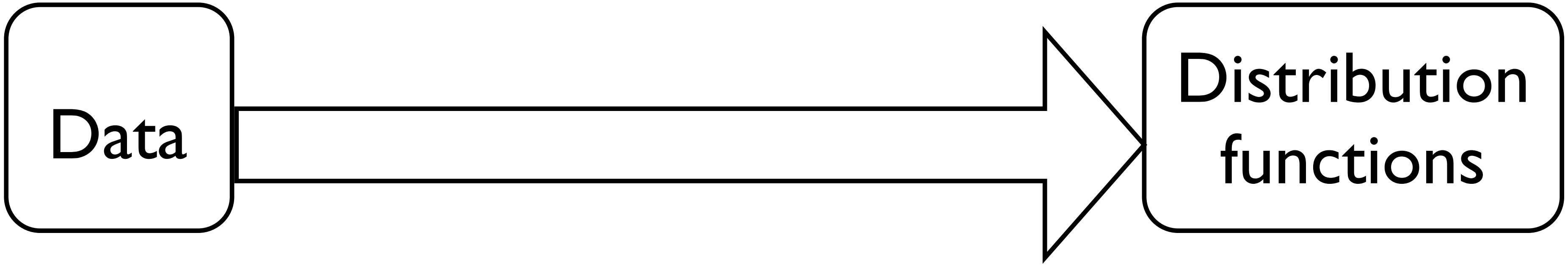

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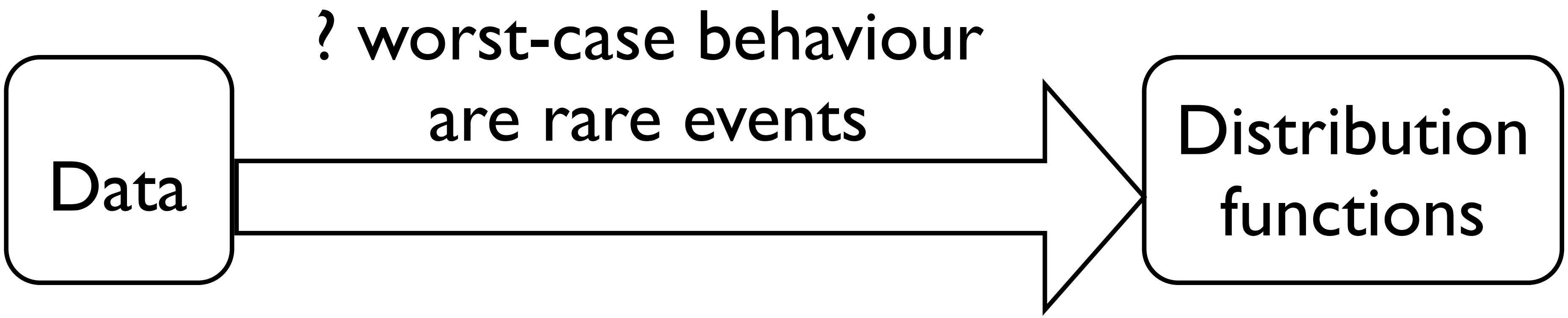

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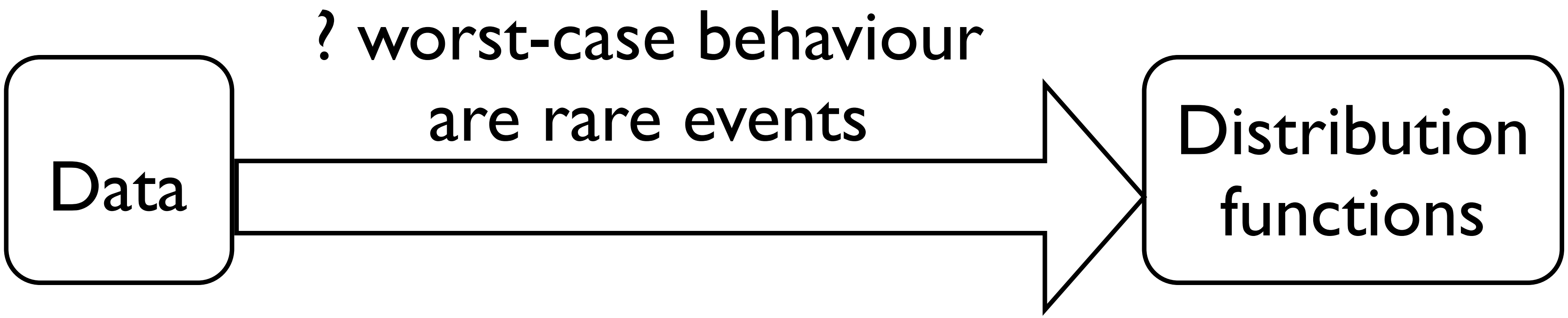

Joint work with N. Navet and René Schott (TRIO, Nancy)

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- ☐ Monte-Carlo simulation
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  - ☑ Large deviation
    - better suited than simulation to rare events
    - easily implementable
    - embedded in a broader analysis

Large deviation : main result

$$M_n = \frac{1}{n}\sum_{k=1}^{n} R_{i,k}$$ mean of response times over n task instances

$$P(M_n \geq value)$$

Cramer's theorem : if $R_{i,n}$ independent identically distributed random variables

$$P(M_n \in \mathbb{G}) \asymp e^{-n \inf_{x\in\mathbb{G}} I(x)}$$

$$\mathbb{G} = [value, \infty)$$

$$I(x) = \sup_{\tau>0}[\tau x - \log E(e^{\tau x})] = \sup_{\tau>0}[\tau x - \log \sum_{k=-\infty}^{+\infty} p_k e^{k\tau}]$$

Technical contribution

Can deal with distributions given as histograms

RT interval	Probability	k
$[0,10)$	$1/25$	5
$[10,20)$	$2/25$	15
$[20,30)$	$3/25$	25
$[30,40)$	$10/25$	35
$[40,50)$	$4/25$	45
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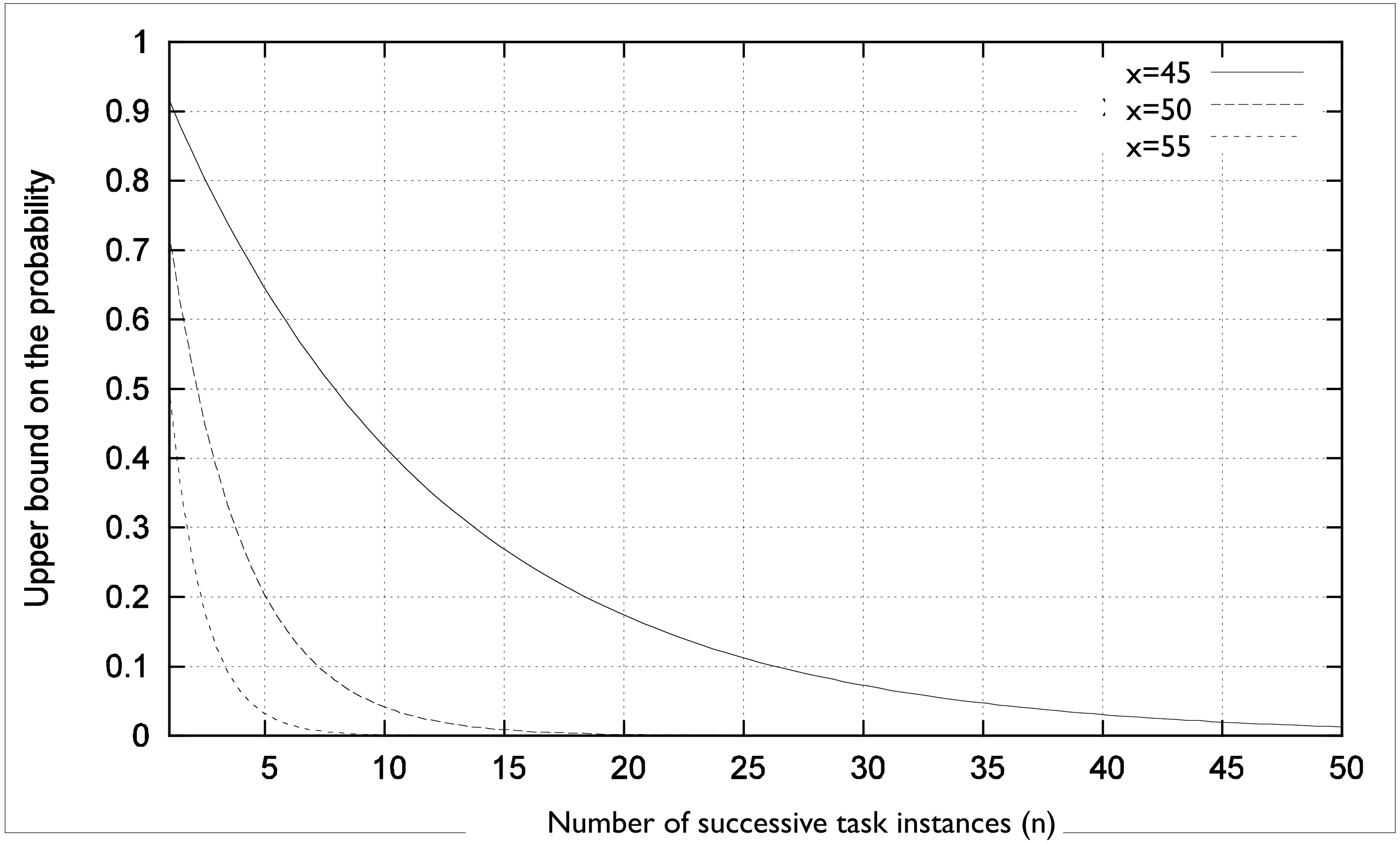

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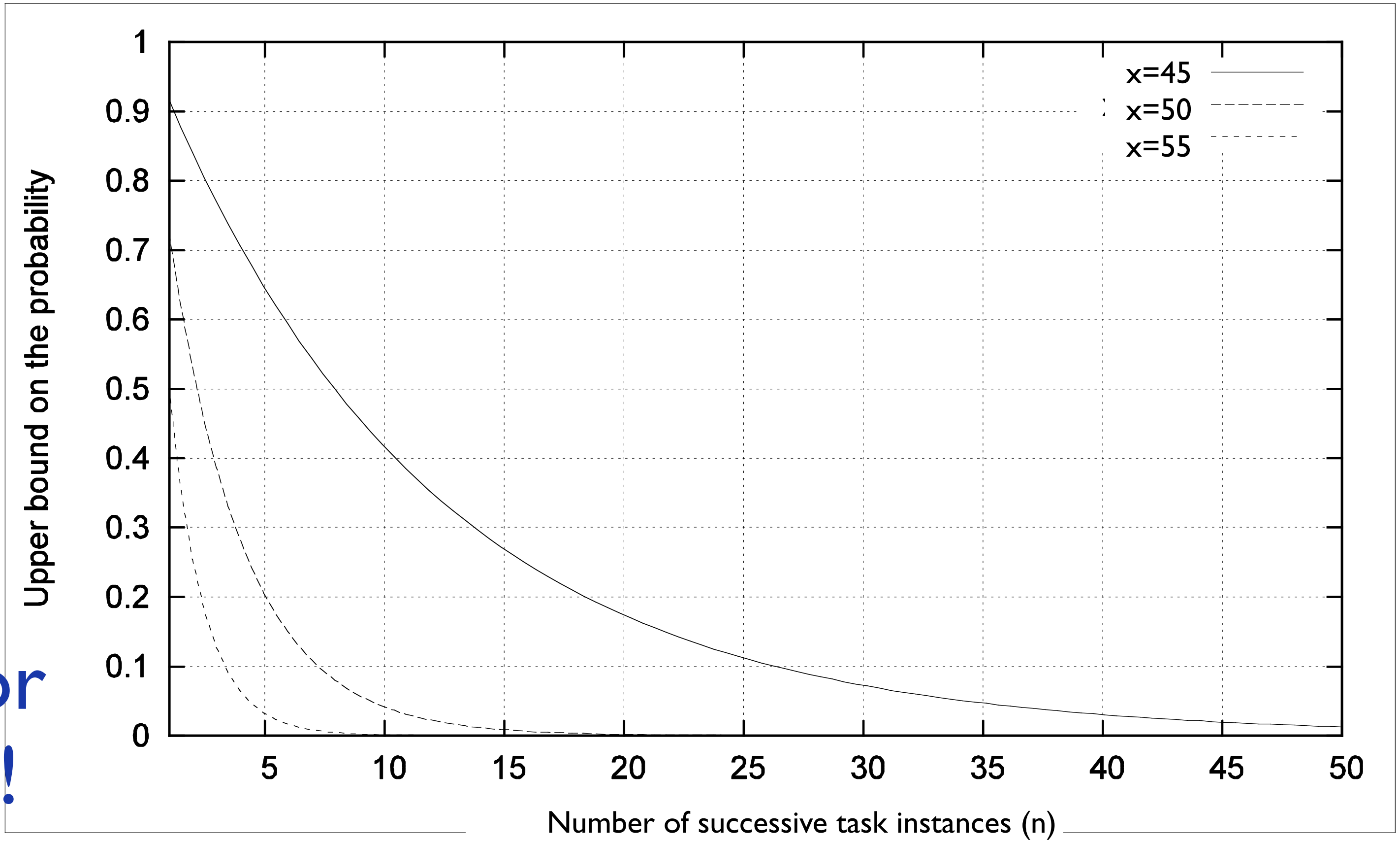

!! Uniprocessor or multiprocessor !!

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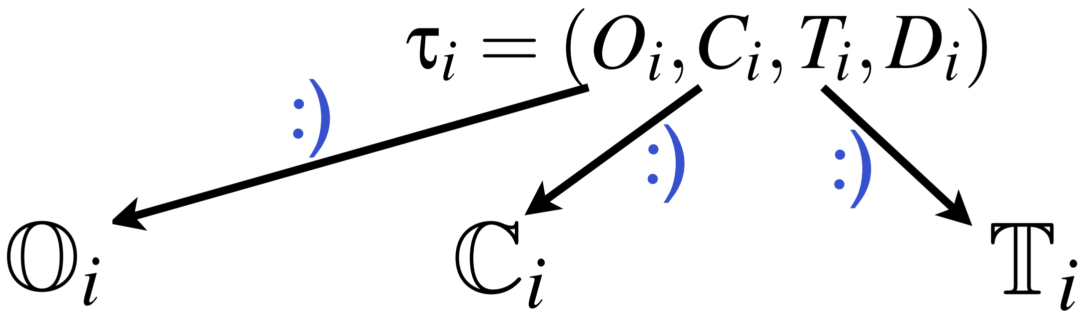

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Joint work with E. Tovar (Hurray, Portugal)

Response time of a task τ_i

When minimal inter-arrival times are considered

$$R_i = C_i + \sum_{j \in hp(i)} \lceil \frac{R_i}{T_j} \rceil C_j$$

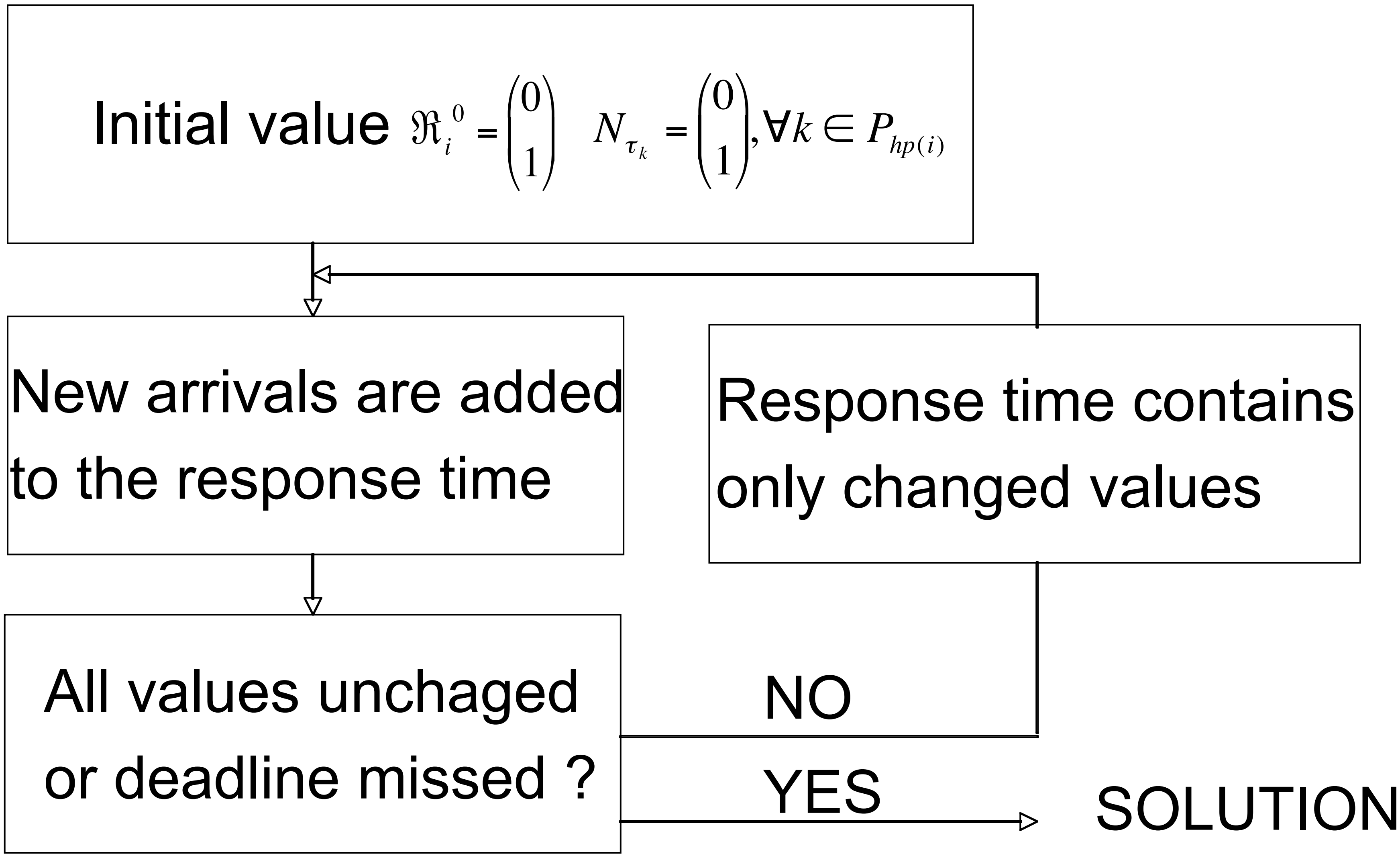
This time ...

Response time

$$\mathbb{R}_i = C_i \otimes (\otimes_{k \in P} \lceil \frac{\mathbb{R}_i}{C_{\ k}}) \otimes (\otimes_{k \in R} N_{\tau_k} C_k)$$

Algorithm providing a solution

$$\mathbb{R}_i = \begin{pmatrix} 6 & 9 & 11 \\ 0.5 & 0.3 & 0.2 \end{pmatrix}$$



Initial values

$$\Re_i^{\ 0} = \begin{pmatrix} r_{i,1}^{0} \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \text{ and } N_k = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \forall k \in R_{hp(i)}$$

Iteration *m* - first step

Working random variable L^m

$$L_j^m = C_i + \sum_{k \in P_{hp(i)}} \left\lceil \frac{L_j^m}{T_k} \right\rceil \cdot C_k + \sum_{k \in R_{hp(i)}} N_k(r_j^{m-1}) \cdot C_k$$

r_j^{m-1} initial value

An example

Task	T	C
τ_1	$T_1 = \begin{pmatrix} 8 & 10 & 15 \\ 0.1 & 0.3 & 0.6 \end{pmatrix}$	3
τ_2	$T_2 = \begin{pmatrix} 10 \\ 1 \end{pmatrix}$	3
τ_3	$T_3 = \begin{pmatrix} 15 & 20 \\ 0.6 & 0.4 \end{pmatrix}$	2
τ_4	$T_4 = \begin{pmatrix} 15 \\ 1 \end{pmatrix}$	2
τ_5	$T_5 = \begin{pmatrix} 14 & 22 \\ 0.4 & 0.6 \end{pmatrix}$	2

$L_4^1 = \begin{pmatrix} l_1^1 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$ where l_1^1 solution of equation $l_1^1 = 0 \cdot C_1 + \left\lceil \frac{l_1^1}{T_2} \right\rceil C_2 + 0 \cdot C_3 + 1 \cdot C_4$

with $r_1^0 = 0$ initial value

Iteration *m* - second step

$$\Re_i^m = L^m \otimes \left(\bigotimes_{k \in R_{hp(i)}} \Delta_k \cdot C_k \right)$$

Back to the example

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τ_5	$T_5 = \begin{pmatrix} 14 & 22 \\ 0.4 & 0.6 \end{pmatrix}$	2

$$\Re_4^2 = L^2 \otimes \begin{pmatrix} 1 & 2 \\ 0.6 & 0.4 \end{pmatrix} C_1 \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} C_3$$

Iteration *m* - get ride of unchanged values

$$\begin{cases} L^m = \begin{pmatrix} 1 & 3 & 4 \\ 0.5 & 0.2 & 0.3 \end{pmatrix} \\ \Re_i^m = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 0.1 & 0.4 & 0.3 & 0.1 & 0.1 \end{pmatrix} \end{cases}$$

$$\text{New } \Re_i^m = Comp(\Re_i^m, L^m) = \begin{pmatrix} 2 & 5 \\ 0.4 & 0.1 \end{pmatrix}$$

One entire iteration (3) of our example

The periodic higher tasks are giving a response time:

$$\mathrm{I} = \begin{pmatrix} 20 \\ 0.5 \end{pmatrix}$$

The random higher tasks are giving a response time:

$$\Re^{3}{}_{n,0} = \mathrm{I} \otimes \left(F^{*}(20) \cdot \mathrm{C}_{3} \right) = \begin{pmatrix} 20 & 21 \\ 0.42 & 0.08 \end{pmatrix},$$

$$\text{where } F^{*}(20) = \begin{pmatrix} 2 & 3 \\ 0.84 & 0.16 \end{pmatrix}$$

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- ☑ Next step?

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- ☑ Robustness condtion: worst-case behavior of the algorithms is rare
- ☑ Worst-case condition: insure worst-case when mixing hard and soft

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M. Pinedo - *Offline deterministic scheduling, stochastic scheduling and online deterministic scheduling: a comparative overview*

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Open problems in
stochastic real-time scheduling :
introduction of dependent
random variables