Optimizing the configuration of X-by-Wire networks using word combinatorics

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TTP/C – Time Triggered Protocol

- Designed at T.U. Vienna + TTTech
- TTP/C main technical characteristics:
  - Determinism
  - Fault-Tolerance
  - Composability
  - Support of mode changes

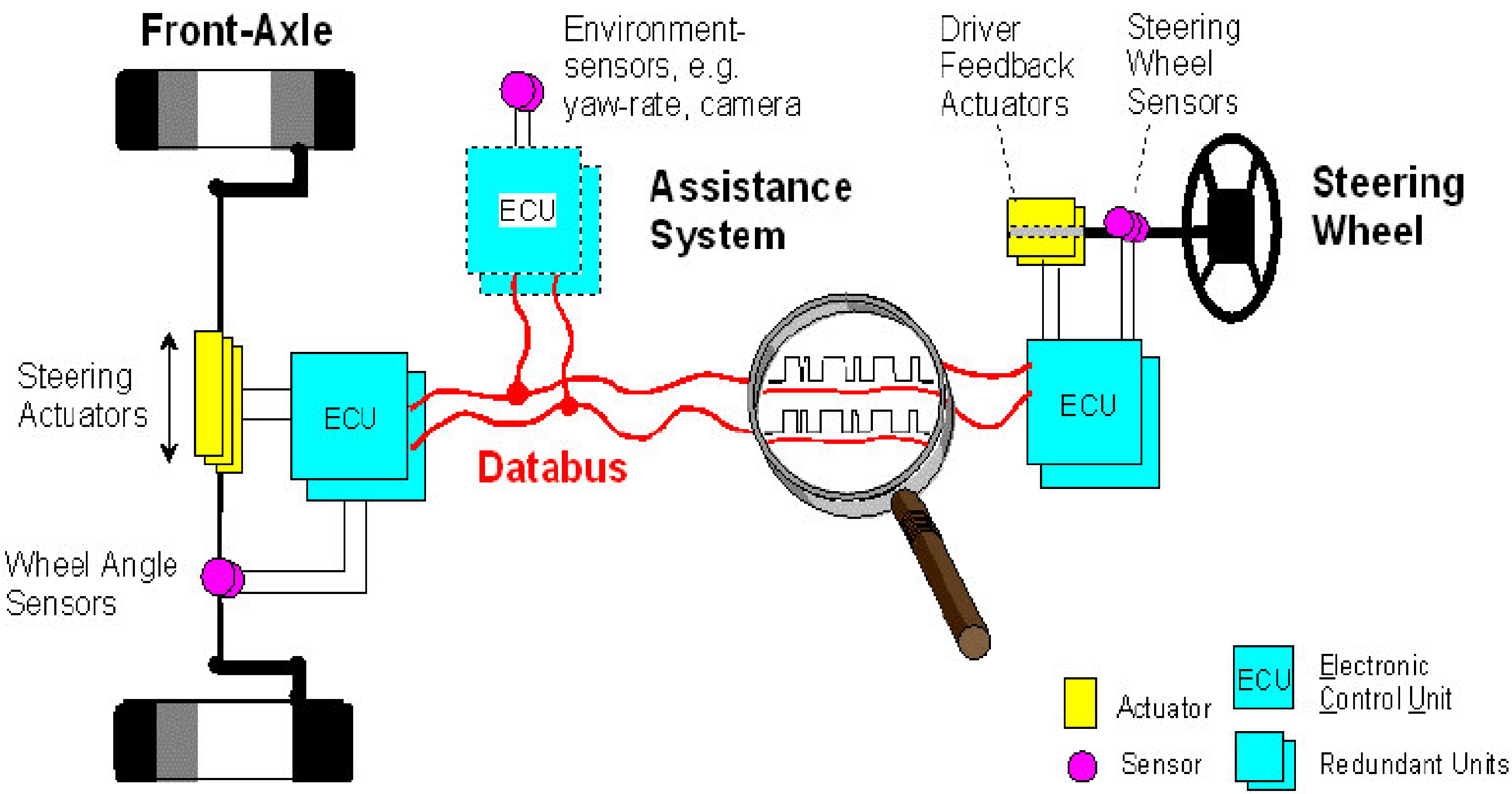
⇨ A good candidate for X-By-Wire ..

X-by-Wire

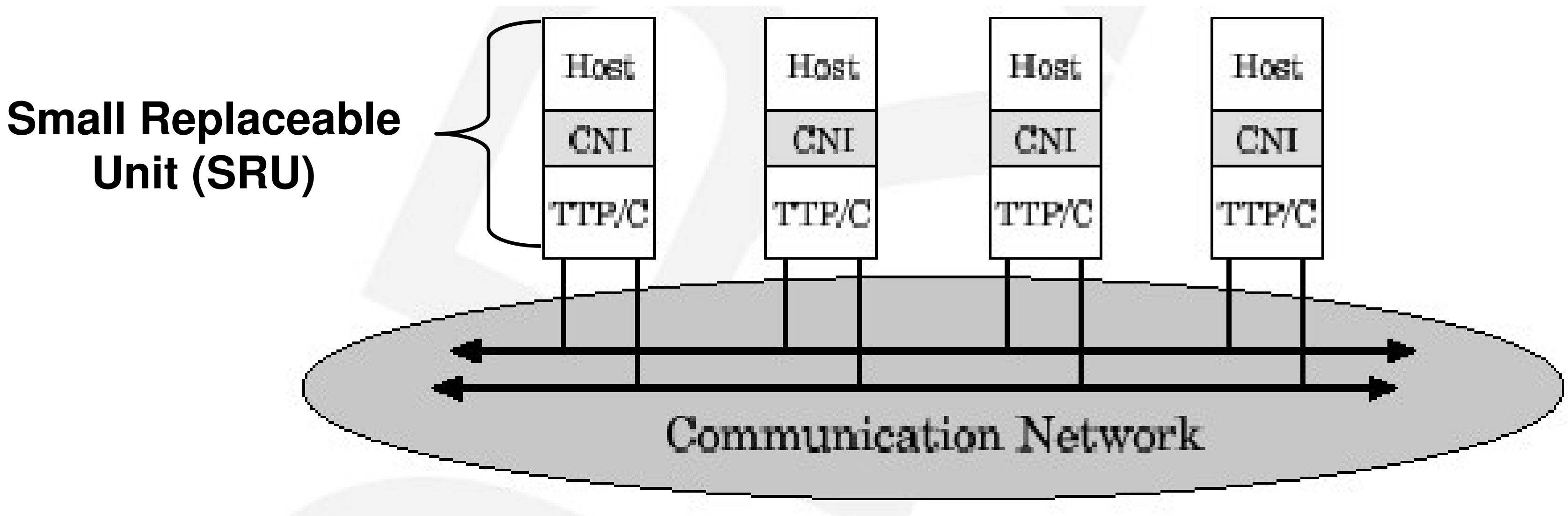
- Hydraulic and mechanical connection are replaced by networks and actuators

- Why ?
  - Decrease of weight and cost
  - Safety : intrusion of the steering column in the cockpit
  - New functions : variable demultiplication - crash avoidance
  - Less pollution (brake / transmission liquid)
  - …

X-by-wire : an example

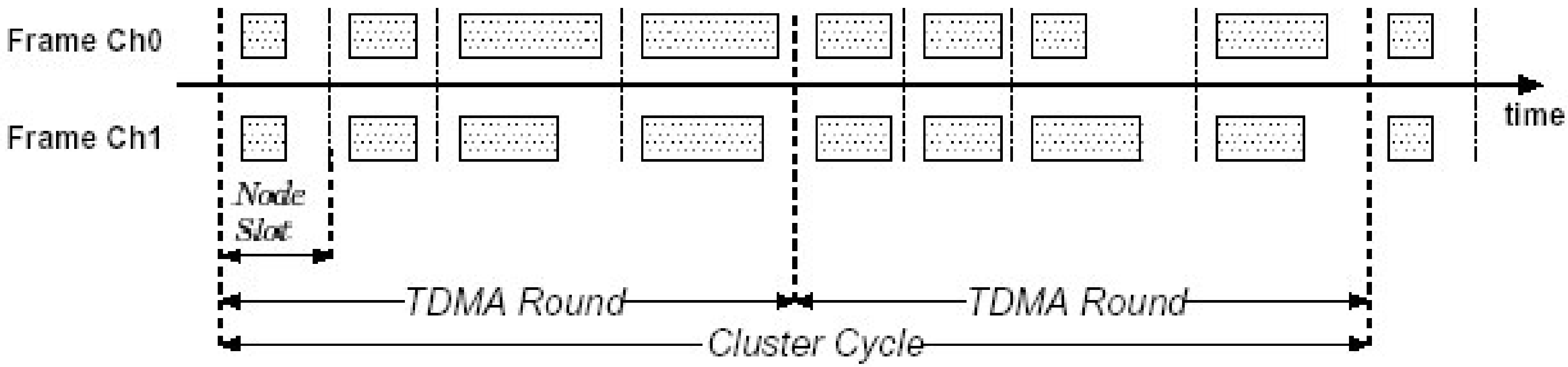


A TTP/C Cluster



- Medium Access Control : TDMA
- Redundant transmission support
- Data rate: 500kbit/s, 1Mbit/s, 2Mbit/s, 5Mbit/s, 25Mbit/s
- Topology: bus or star

TDMA – Time division Multiplexed Access



- Slot: time window given to a station for a transmission
- TDMA Round: sequence of slots s.t. each station transmits exactly once
- Cluster Cycle: sequence of the ≠ TDMA rounds

TTP/C: Implications of the MAC protocol

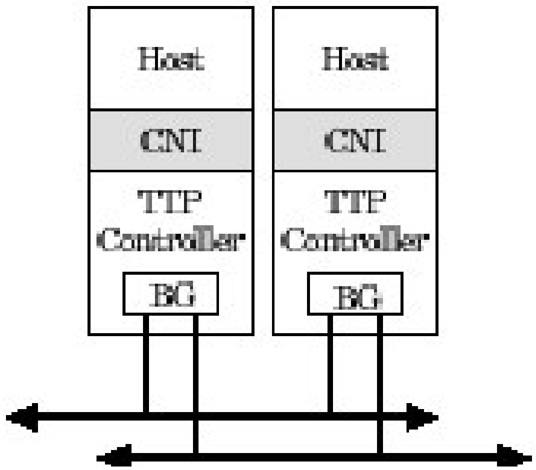
Bounded response times and « heartbeats » but:

- loss of bandwidth
- need of powerful CPU's
- maximum timing contraint:
  - If a station sends a single information, the refresh cannot be more frequent than the length of a round
  - If a station sends several informations, the refresh cannot be more frequent than 2x the length of a round

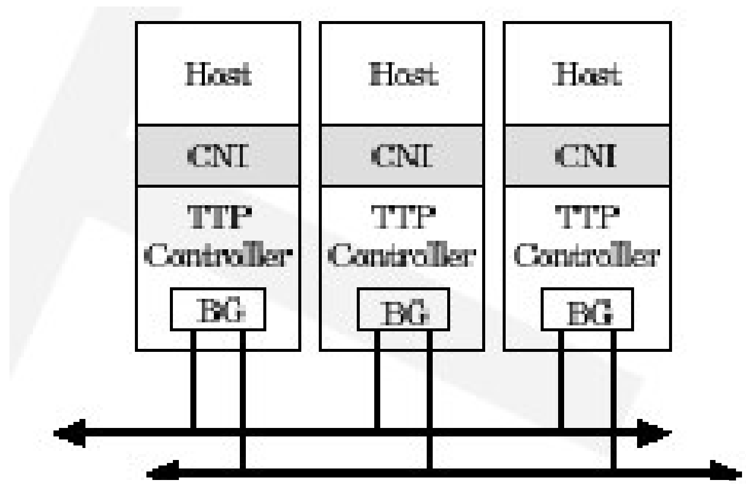
Ex: 5ms time constraint - 500kbit/s network with 200 bits per frames - at most 12 frames (6 FTUs of two nodes) or 6 frames if the station sends 2 distinct informations

FTU: Fault Tolerant Unit

- FTU = set of stations that act identically



FTU of cardinality 2



FTU of cardinality 3
(majority vote is possible)

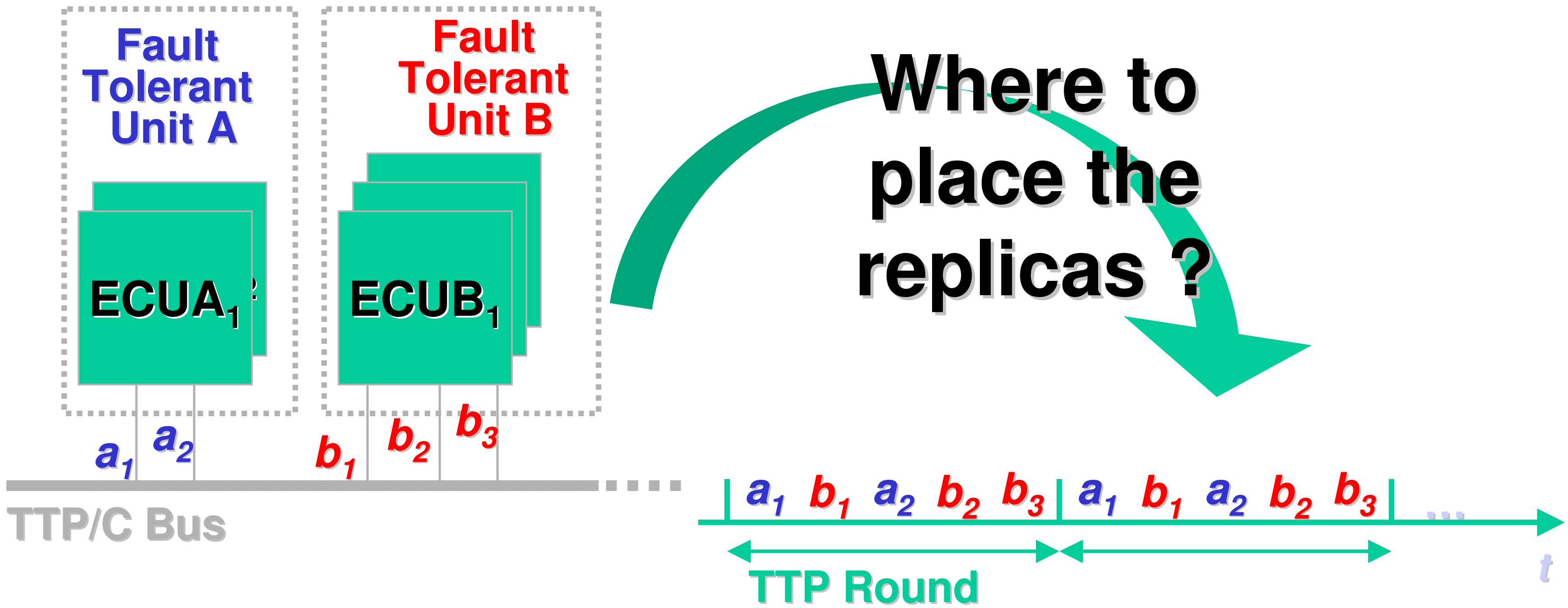
- Replica = a frame sent by a node of the FTU

FTU: which protection ?

- Protection against:
  - disappearance of a station (crash, disconnection..)
  - corrupted frames (EMI)
  - sensors or computation errors
  - …
- Under the assumption of a single failure (TTP/C fault-hypothesis) :
  - A dual redundancy ensures a protection in « the temporal domain »
  - A triple redundancy ensures in addition a protection in « the value domain »
- Problem: history-state

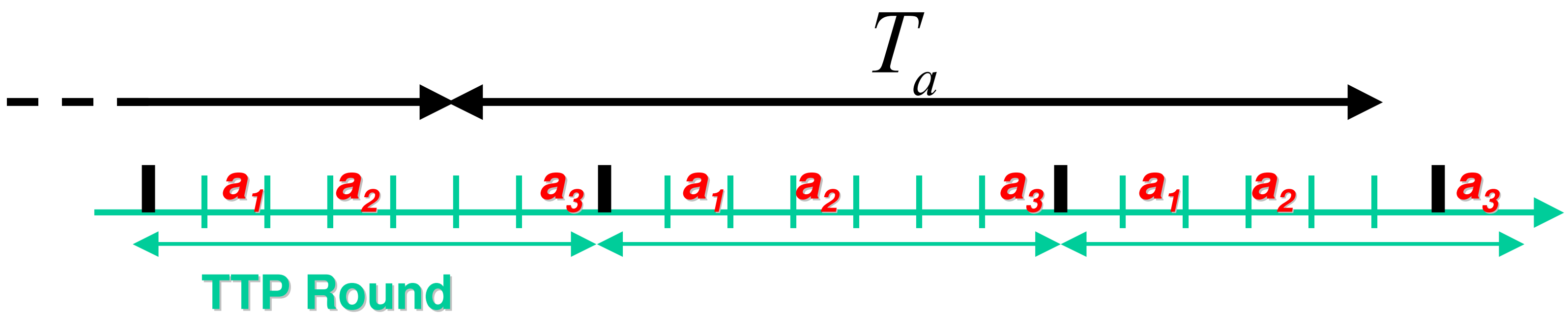
Goal of the study: maximize the robustness against transmission errors

- Transmission errors are usually highly correlated



Application model

- T_a: production cycle of the data sent by the stations of the FTU a



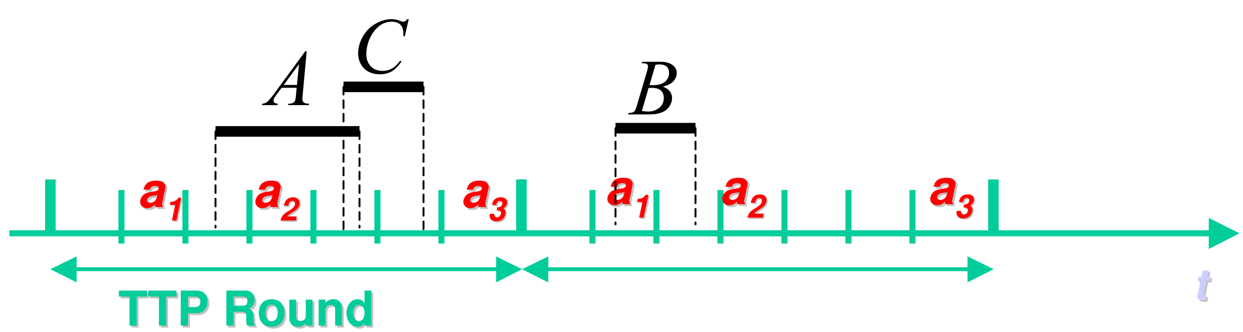
- Assumptions:
  - no synchronization between production and transmission (round)
  - production cycle is a multiple of the length of a round

Objective w.r.t. fail-silence

- A node is « fail-silent » if one can safely consume its data when the frame carrying the data is syntaxically correct
- Stations are fail-silent: « minimize P*all* : the probability that all frames of the FTU sent during a production cycle will be corrupted »
- Stations are not fail-silent: « minimize P*one* : the probability that at least one frame of the FTU will be corrupted»

Assumptions on the error model

- Each bit transmitted during an EMI will be corrupted with probability π
- If a perturbation overlaps a whole slot, the corresponding frame is corrupted with probability 1
- Starting times of the EMI bursts are independent and uniformly distributed over time
- The distribution of the size of the bursts is arbitrary



Objective 1 : Minimize P*one*

Majorization - Schur-Convexity

- vector $u = (u_1, \ldots, u_n)$ majorizes $v = (v_1, \ldots, v_n)$ if:

$$\sum_{i=1}^{n} u_i = \sum_{i=1}^{n} v_i \text{ and } \sum_{i=1}^{k} u_{[i]} \leq \sum_{i=1}^{k} v_{[i]} \quad k \leq n$$

with $(u_{[i]}, \ldots, u_{[n]})$ permutation of u s.t. $u_{[i]} \leq \ldots \leq u_{[n]}$

Example: $(1,3,5,10) \succ (2,4,4,9)$

- A fonction $f : \Re^n \rightarrow \Re$ is

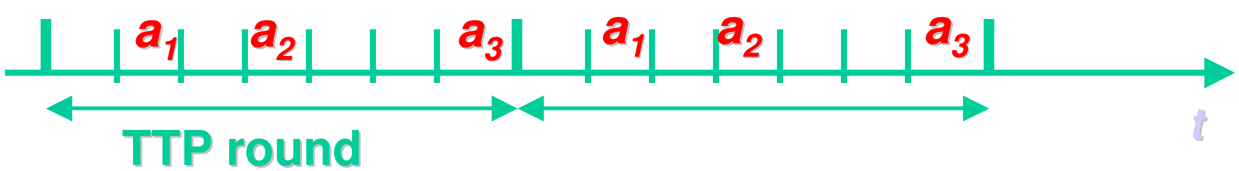
Schur-convex if $u \succ v \rightarrow f(u) \geq f(v)$

Schur-concave if $u \succ v \rightarrow f(u) \leq f(v)$

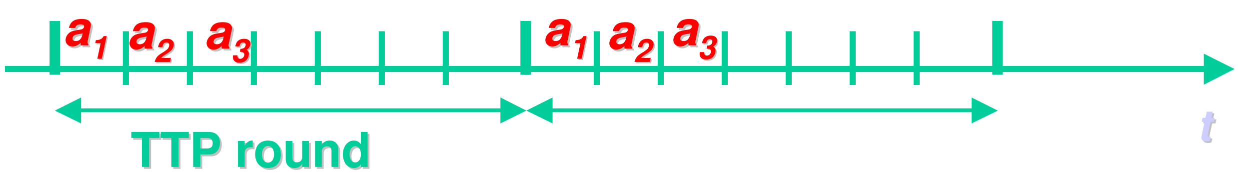
Minimizing P_{one}

- $\mathbf{I}(x)$ is the vector of the distance between replicas during the length of a round (sorted in ascending order)

Example: $\mathbf{I}(x) = (1, 1, 2)$



Example: $\mathbf{I}(x) = (0, 0, 4)$



Minimizing P_{one}

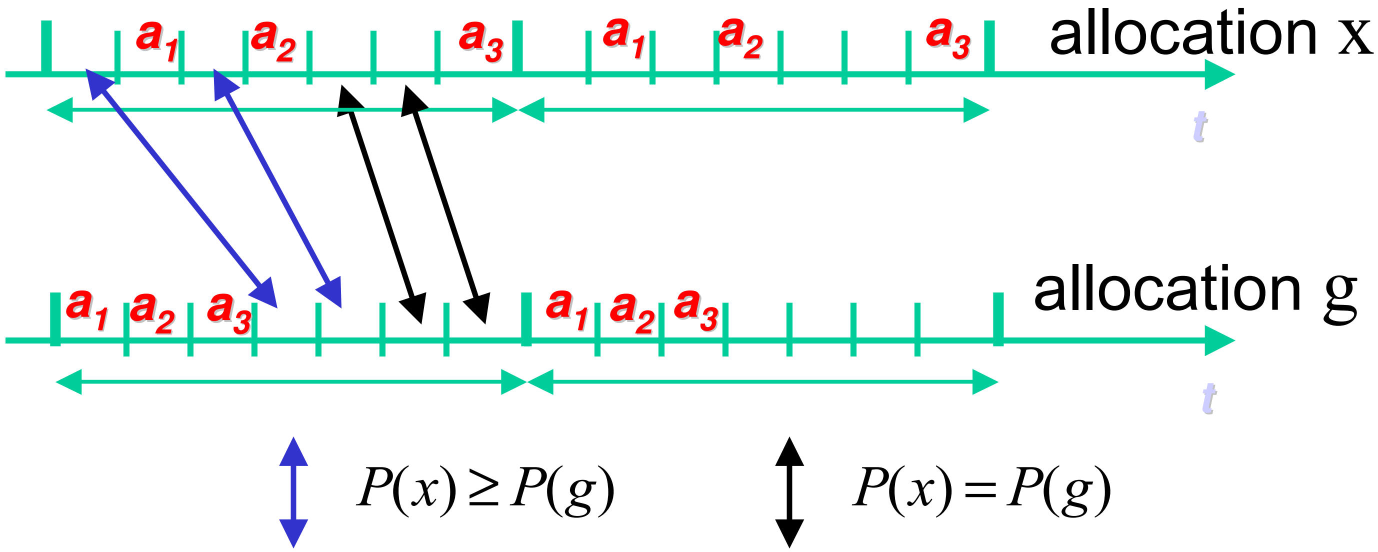
Theorem: the best allocation for P*one* is to group together all replicas (denoted allocation g)

Arguments:

- P*one* is shur-concave: $\mathbf{I}(x') \succ \mathbf{I}(x) \rightarrow P_{one}(x') \le P_{one}(x)$
- $\mathbf{I}(g)$ is maximum for the majorization (equal to $(0,0,...,S-k)$ with k the number of replicas of the FTU and S the number of slots per round)

Minimize P*one*

- Idea of the proof (step 1): the farther the beginning of an error burst from a replica, the less likely the replica becomes corrupted. « Non-grouped » allocations have more areas close to replicas



$P(x) \geq P(g)$ $\quad$ $P(x) = P(g)$

Minimize P*one*

- Validity of the result :
  - Arbitrary π value and burst size distribution
  - Production period multiple of the round length
  - for all TDMA networks

➢ Combined minimization of P*one* for all FTU's is possible

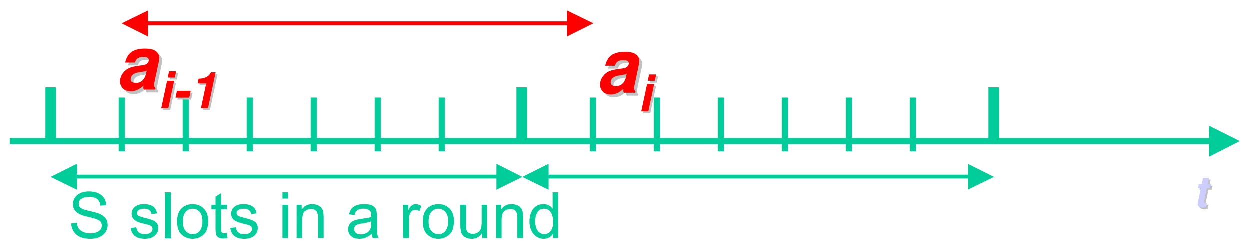
➢ Robustness improvement: against a random allocation, the number of lost data is reduced from 15 to 20% on average

Objective 2 : Minimize P*all*

TTP/C case

TTP/C : the majority rule

- Cliques: sets of stations that disagree on the state of the network
- Principle: to avoid cliques, stations in the minority disconnect (« freeze »)
- Mechanism: before sending, a station checks that in the last round (S slots), the number of correct messages is greater than the number of incorrect messages, otherwise it disconnects



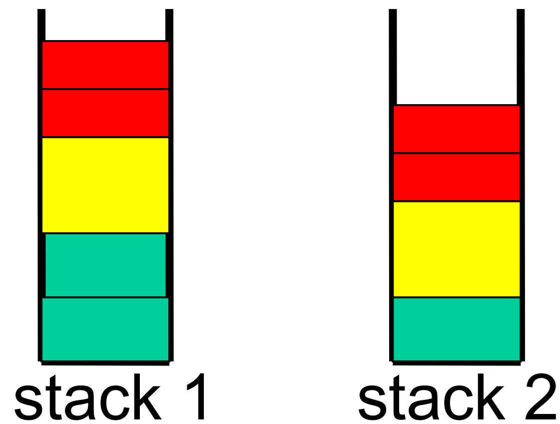
➢ If a station « freezes » due to transmission errors, the others follow one by one...

TTP/C : minimize P*all*

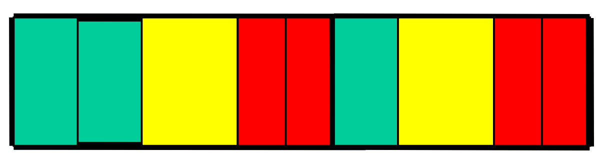
Algorithm: 1) for each FTU i with C_i slots, push $\lceil C_i / 2 \rceil$ slots in the smallest stack and $\lfloor C_i / 2 \rfloor$ in the largest stack

2) concatenate the two stacks

Ex: FTU A: 3 replicas – FTU B: 2 replicas – FTU C: 4 replicas



TTP/C round :



TTP/C : minimize P*all*

Theorem: the « 2-stacks » algorithm is optimal under TTP/C

Arguments:

Case 1) a perturbation for each replica : identical ∀ allocation

Case 2) a perturbation can corrupt several replicas with a probability decreasing in the distance between the replicas. A burst of more than $\lfloor S/2 \rfloor$ slots freezes the system, now the algorithm ensures a distance of $\lfloor S/2 \rfloor$ slots

Corollary: it is useless to have more than 2 replicas per FTU if the probability to have more than one perturbation in the same round is sufficiently low

Objective 2 : Minimize P*all*
TDMA case

Balanced words

- A « balanced » word (or Sturmian word) is a binary sequence $\{u_n\}_{n\in\mathbb{N}}$ s.t. :

$$\forall k,n,m \in \mathbb{N} \quad \left| \sum_{i=n}^{n+k} u_i - \sum_{j=m}^{m+k} u_j \right| \le 1$$

- Balanced words are computed using bracket sequences :

$$u_n = \left\lfloor n\frac{a}{b} \right\rfloor - \left\lfloor (n-1)\frac{a}{b} \right\rfloor$$

where a/b is the rate of the word (nb of 1 / nb of 0)

- Example: balanced word of rate 3/8

$$(0,0,1,0,0,1,0,1)$$

Multimodular functions

- Multimodularity [Hajek] : counterpart of convexity for discrete functions $f : \mathbb{Z}^m \rightarrow \mathbb{R}$

Definition : Let *F* be a set of m+1-vectors that sum to 0, a function $f : \mathbb{Z}^m \rightarrow \mathbb{R}$ is *F*-multimodular if

$$f(x+v)+f(x+w) \geq f(x)+f(x+v+w)$$

$$x \in \mathbb{Z}^m \text{ et } v, w \in F,\ v \neq w$$

- Example: $x = (0,1,0,\ldots,1,1,0)$ is a control sequence, f a cost function and v an elementary operation moving a client to the left

$$v = (0,\ldots,0,1,-1,0,\ldots,0)$$

Optimisation and multimodular function

- Global left shift operator : $s_i(x)$

$$\text{ex: } s_2\big((0,1,0,1,1,0)\big) = (0,1,1,0,0,1)$$

Theorem [Altman,Gaujal,Hordijk 97]: If *f* is multimodular then $G(x) = 1/m\sum_{i=1}^{m} f(s_i(x))$ (shift-invariant version of *f*) is minimum (among all admissible sequences) if x is a balanced sequence.

Theorem : If the size of the bursts is exponentially distributed then P*all* is multimodular.

Moreover, P*all* is equal to its shift invariant version thus P*all* is minimum for a balanced sequence.

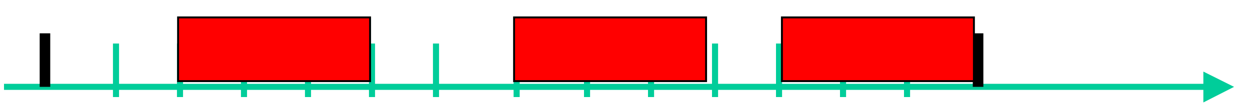
Optimal algorithm : a single FTU

- FTU with C replicas of size h bits in a round of total size R bits
  - compute v_i balanced word of rate $C/(R - C(h-1))$
  - x is the round initially empty
  - If $v_i = 1$ then $x := x + 1...1$ (h '1' concatenated)
  - If $v_i = 0$ then $x := x + 0$

Ex: FTU: 3 replicas of cardinality 3 in a round of size 14

$v_i = (0,0,1,0,0,1,0,1)$ with rate $3/8$

$x = (0,0,1,1,1,0,0,1,1,1,0,1,1,1)$ ($\neq$ balanced word with rate $9/14$)



Optimal algorithm : several FTUs

- Problem : allocation conflicts

  Ex: FTU A: 3 replicas – FTU B: 2 replicas – FTU C: 1 replica

$$x_A = (0,1,0,1,0,1) \text{ rate } 3/6$$
$$x_B = (0,0,1,0,0,1) \text{ rate } 2/6$$
$$x_C = (0,0,0,0,0,1) \text{ rate } 1/6$$

- An optimal allocation is still possible if
  - the number of cardinalities is a power of 2
  - all replicas have the same size

- Remark: a balanced sequence is minimum for the majorization order, it is thus the worst solution for P*one*! Both objectives are contradictory

Heuristic : several FTUs

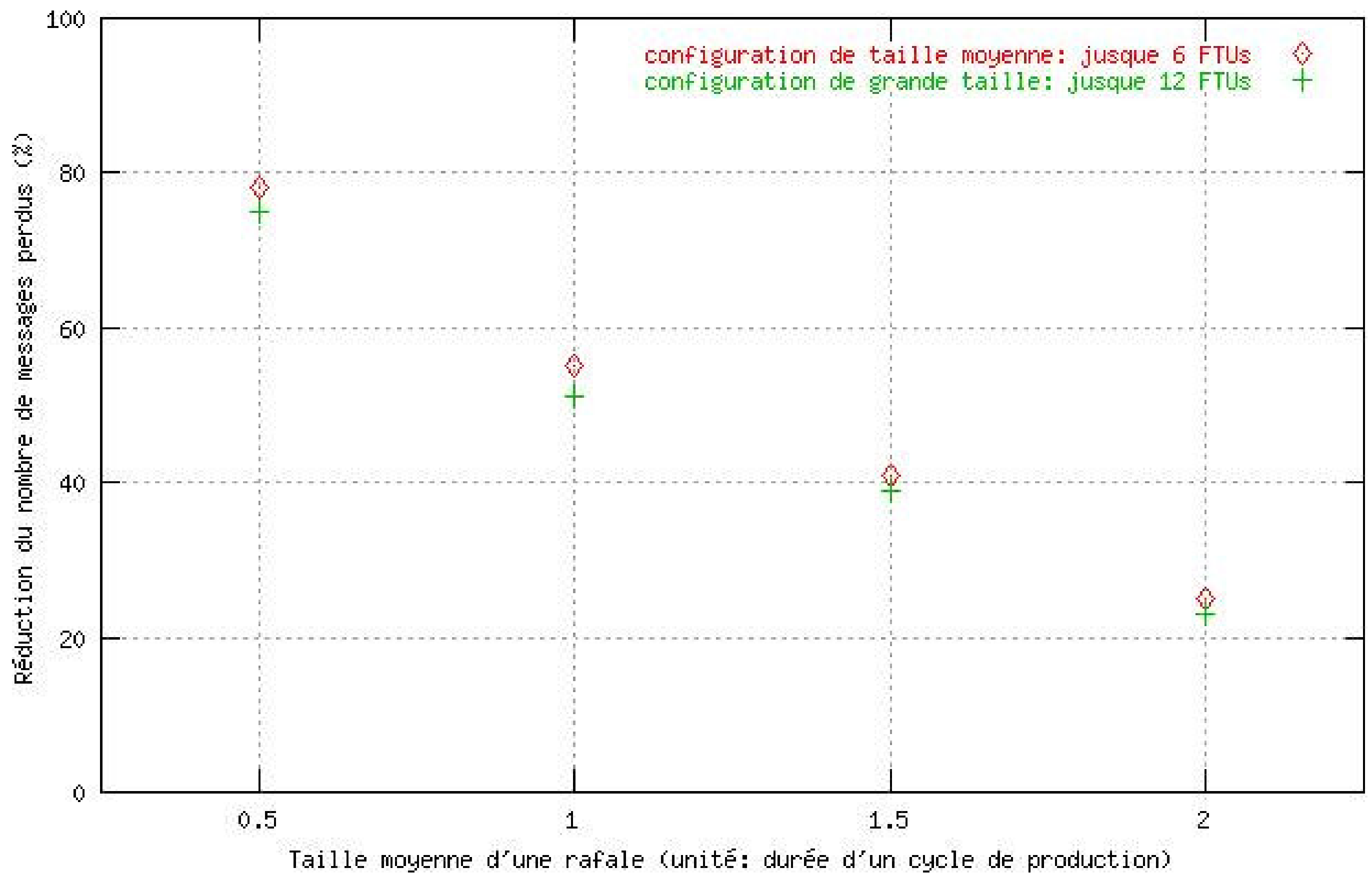
- The rate of emission: number of bits that FTU A must emit on average during each bit of a round :

$$d_A = C_A h_A / R$$

- At step i, one schedules the transmission of a frame of the FTU for which the number of due bits – number of already allocated bits is maximum

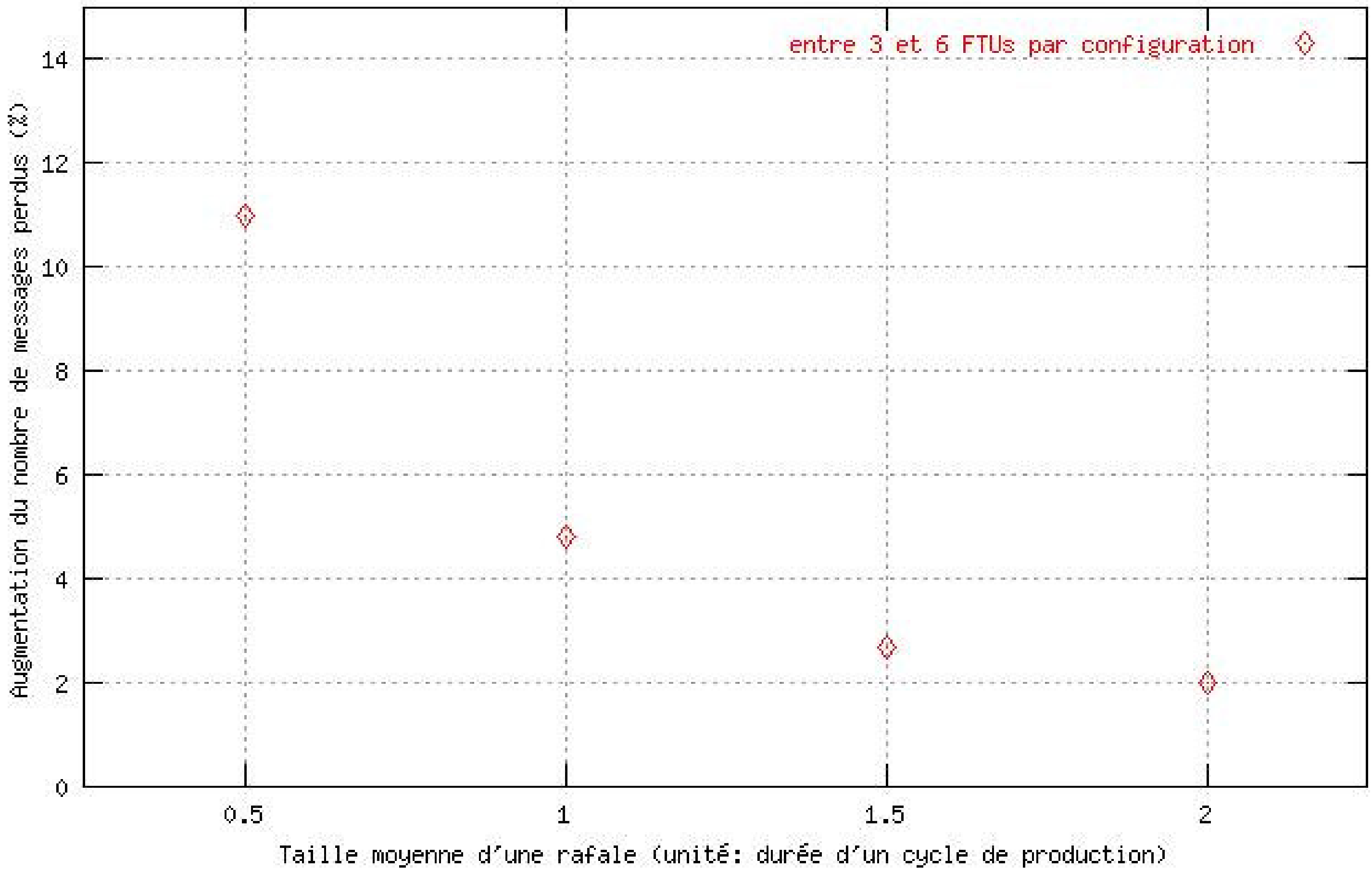
P*all* : Heuristic vs random allocation

- Reduction of the number of lost messages w.r.t. a random allocation :



P*all* : Heuristic vs optimal

- Increase of the number of lost messages w.r.t. to the optimal :



Conclusion

Optimal and near optimal allocation policies for TDMA and TTP/C networks

- Choice of the locations of the slots have a strong influence on the robustness of the network
- The cost function plays a major role on the shape of the solution
- Hypothesis on the error model are crucial

Future work :

- Configurations made of fail-silent and non fail-silent nodes (minimizing P*one* and P*all* for different FTU's)
- FlexRay protocol