Mixed Time Frameworks for the Periodically Aggregated Resource-Constrained Project Scheduling Problem

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Journées GOThA/Bermudes 2017

École Polytechnique de l'Université de Tours

Tuesday 26th September, 2017

- Linear formulations for resource-constrained scheduling problems
  - ILP formulations based on time-indexed variables
  - Introduction of continuous time variables (event-based)
- Problems such that:
  - Resource usage is evaluated on average in periods of parameterized length
  - Precise handling of activity start and completion times

C. Artigues, M. Gendreau, L.-M. Rousseau, A. Vergnaud.
Solving an integrated employee timetabling and job-shop scheduling problem via hybrid branch-and-bound.
In: Computers & Operations Research 36 (8) pp. 2330–2340, 2009.

- RCPSP/π: more general, purely discrete problem

J. Böttcher, A. Drexl, R. Kolish, F. Salewski.
Project Scheduling Under Partially Renewable Resource Constraints.
In: Management Science 45 (4) pp. 543–559, 1999.

Input

- $\mathcal{A}$ Set of n activities
- $\mathcal{R}$ Set of m renewable resources
- p_i Processing time of activity $i \in \mathcal{A}$
- b_k Capacity of resource $k \in \mathcal{R}$
- $r_{i,k}$ Request of activity $i \in \mathcal{A}$ on resource $k \in \mathcal{R}$
- $E \subset \mathcal{A} \times \mathcal{A}$; end-to-start precedence relations
- Δ Period length

Constraints

1. Precedence constraints
2. Periodically aggregated resource constraints

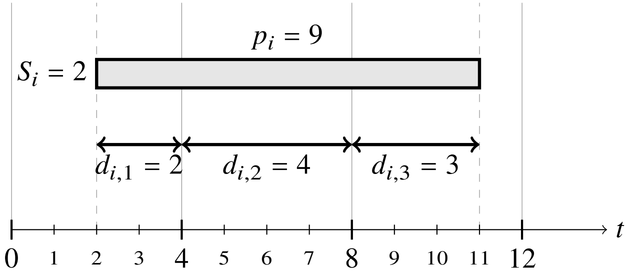
Objective

Minimise the project duration

Solution representation

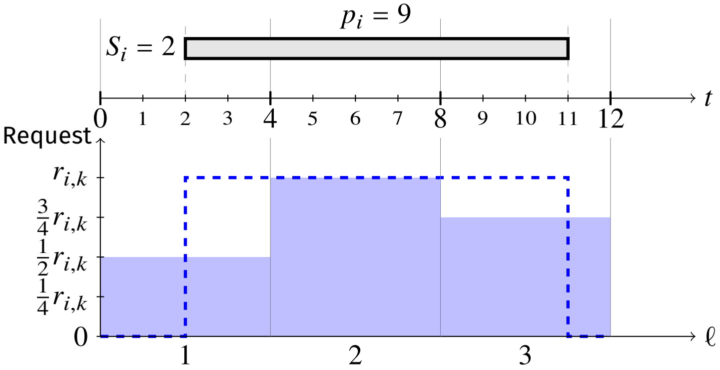
S_i Start date of activity $i \in \mathcal{A}$

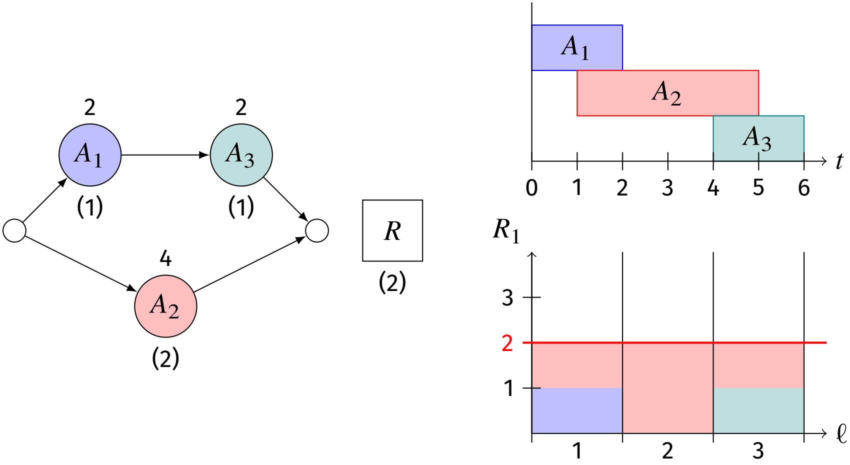
$d_{i,\ell}$ Length of $[(\ell-1)\Delta, \ell\Delta] \cap [S_i, S_i + p_i]$

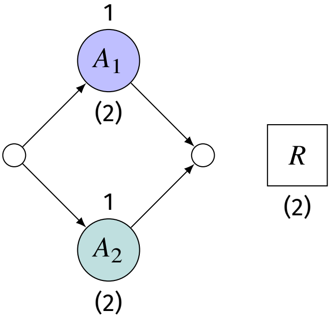


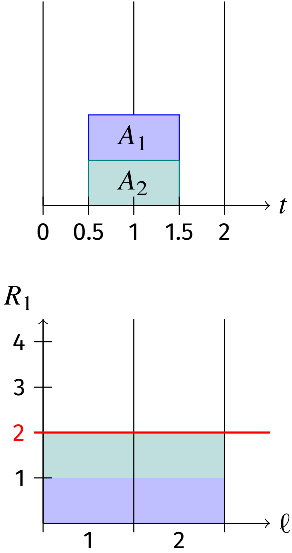
Abstract formulation

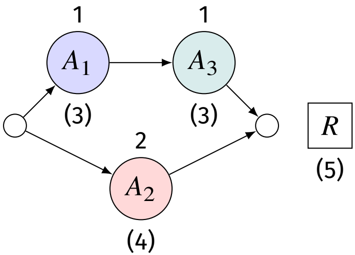
$$
\begin{aligned}
\text{Minimise} \quad & S_{n+1} - S_0 \\
\text{s.t.} \quad & S_{i_2} - S_{i_1} \geq p_{i_1} && \forall (i_1, i_2) \in E \\
& \sum_{i \in \mathcal{A}} r_{i,k} \frac{d_{i,\ell}}{\Delta} \leq b_k && \forall k \in \mathcal{R}, \; \forall \ell \in \mathbb{Z}
\end{aligned}
$$

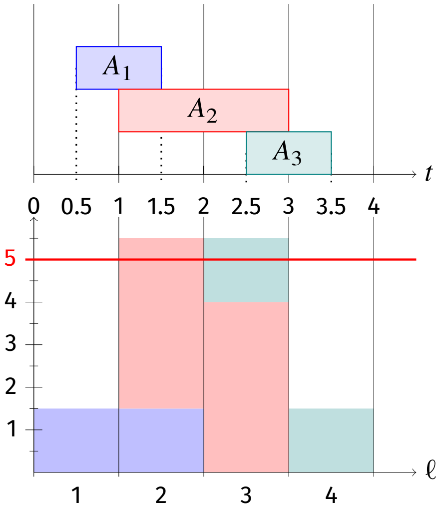



2
A_1
(1)
2
A_3
(1)
4
A_2
(2)
R
(2)
A_1
A_2
A_3
0
1
2
3
4
5
6
t
R_1
3
2
1
1
2
3
ℓ


1
A_1
(2)
1
A_2
(2)
R
(2)


A_1
A_2
t
0
0.5
1
1.5
2
R_1
4
3
2
1
ℓ
1
2


1
A_1
(3)
1
A_3
(3)
2
A_2
(4)
R
(5)


A_1
A_2
A_3
t
0
0.5
1
1.5
2
2.5
3
3.5
4
5
4
3
2
1
ℓ
1
2
3
4

Key points

- PARCPSP = RCPSP relaxation
- Substantial project duration reduction (even with small values of Δ)
- Shifting a schedule may alter its resource-feasibility
  (even with constant resource capacities over time)
- $0 \leq S_0 < \Delta \quad \Rightarrow \quad S_{n+1} - S_0 \leq S_{n+1} = C_{\max}$

P.-A. Morin, C. Artigues, A. Haït, T. Kis, F. Spieksma.
Structural Properties and Complexity of the PARCPSP.
Technical report, LAAS-CNRS (University of Toulouse, France), 2017.

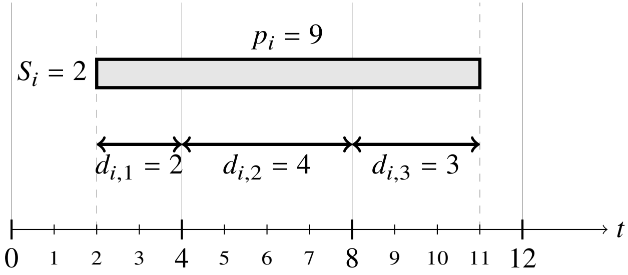
Theorem

The PARCPSP is NP-complete.

Additional data for models

$\mathscr{L}$ Set of L consecutive aggregated periods (length Δ) starting from $t = 0$

Mixed time representation management



How to link S_i and $d_{i,\ell}$?

First mixed time framework (MTF-1)

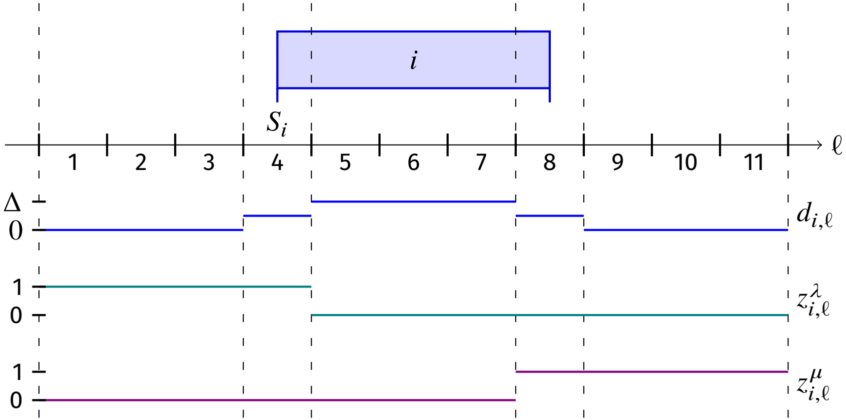
Variables

S_i Start date of activity $i \in \mathscr{A}$

$d_{i,\ell}$ Length of $[(\ell-1)\Delta, \ell\Delta] \cap [S_i, S_i + p_i]$

$z^{\lambda}_{i,\ell}$ $= 1$ if activity $i \in \mathscr{A}$ starts in period $\ell \in \mathscr{L}$ or after, 0 otherwise

$z^{\mu}_{i,\ell}$ $= 1$ if activity $i \in \mathscr{A}$ completes in period $\ell \in \mathscr{L}$ or before, 0 otherwise



i
S_i
1 2 3 4 5 6 7 8 9 10 11
ℓ
Δ
0
$d_{i,\ell}$
1
0
$z^{\lambda}_{i,\ell}$
1
0
$z^{\mu}_{i,\ell}$

Step behavior of $z^{\lambda}_{i,\ell}$ and $z^{\mu}_{i,\ell}$

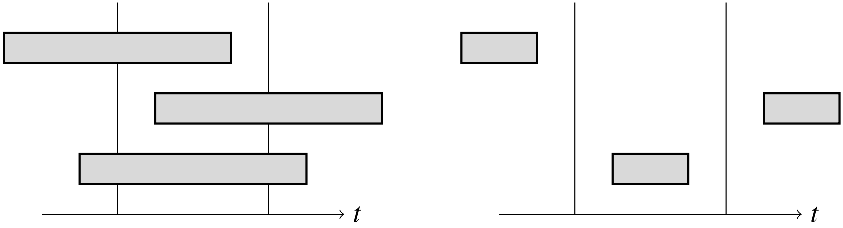
$$z^{\lambda}_{i,\ell+1} \leq z^{\lambda}_{i,\ell} \qquad \forall i \in \mathscr{A}\,,\ \forall \ell \in \mathscr{L}$$
$$z^{\lambda}_{i,1} = 1 \qquad \forall i \in \mathscr{A}$$
$$z^{\mu}_{i,\ell-1} \leq z^{\mu}_{i,\ell} \qquad \forall i \in \mathscr{A}\,,\ \forall \ell \in \mathscr{L}$$
$$z^{\mu}_{i,L} = 1 \qquad \forall i \in \mathscr{A}$$

Bounding S_i

$$S_i \leq (\ell - 1)\Delta + \mathscr{M}\, z^{\lambda}_{i,\ell} \qquad \forall i \in \mathscr{A}\,,\ \forall \ell \in \mathscr{L}$$
$$S_i \geq (\ell - 1)\Delta - \mathscr{M}(1 - z^{\lambda}_{i,\ell}) \qquad \forall i \in \mathscr{A}\,,\ \forall \ell \in \mathscr{L}$$
$$S_i + p_i \leq \ell\Delta + \mathscr{M}(1 - z^{\mu}_{i,\ell}) \qquad \forall i \in \mathscr{A}\,,\ \forall \ell \in \mathscr{L}$$
$$S_i + p_i \geq \ell\Delta - \mathscr{M}\, z^{\mu}_{i,\ell} \qquad \forall i \in \mathscr{A}\,,\ \forall \ell \in \mathscr{L}$$

Computing $d_{i,\ell}$

$$
\begin{aligned}
&d_{i,\ell} \geq \Delta(1 - z^{\lambda}_{i,\ell} - z^{\mu}_{i,\ell}) && \forall i \in \mathcal{A}, \forall \ell \in \mathcal{L} \\
&d_{i,\ell} \leq \Delta(1 - z^{\lambda}_{i,\ell+1} - z^{\mu}_{i,\ell-1}) && \forall i \in \mathcal{A}, \forall \ell \in \mathcal{L} \\
&d_{i,\ell} \geq (\ell\Delta - S_i) - \Delta\, z^{\mu}_{i,\ell} - \mathcal{M}(1 - z^{\lambda}_{i,\ell}) && \forall i \in \mathcal{A}, \forall \ell \in \mathcal{L} \\
&d_{i,\ell} \geq (S_i + p_i - (\ell - 1)\Delta) - \Delta\, z^{\lambda}_{i,\ell} - \mathcal{M}(1 - z^{\mu}_{i,\ell}) && \forall i \in \mathcal{A}, \forall \ell \in \mathcal{L} \\
&\sum_{\ell \in \mathcal{L}} d_{i,\ell} = p_i && \forall i \in \mathcal{A}
\end{aligned}
$$



Second mixed time framework (MTF-2)

Variables

S_i Start date of activity $i \in \mathscr{A}$

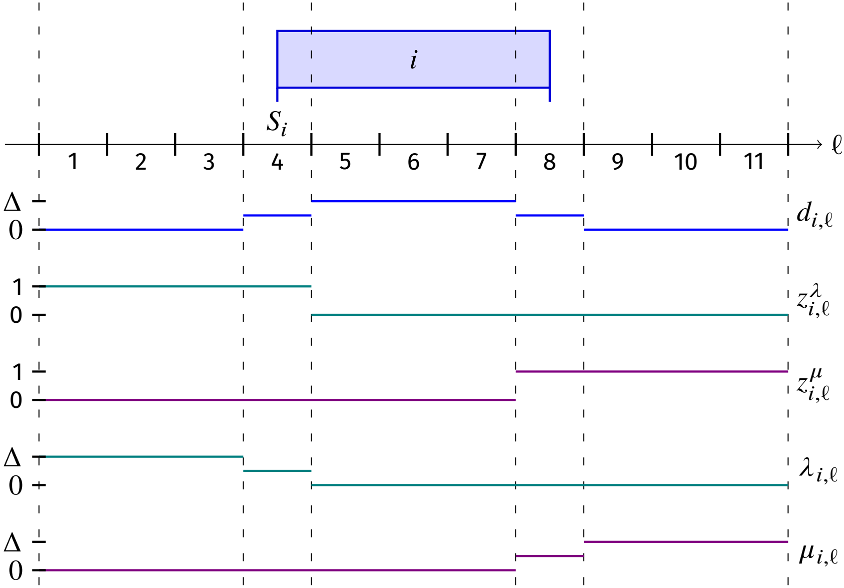
$d_{i,\ell}$ Length of $[(\ell-1)\Delta, \ell\Delta] \cap [S_i, S_i + p_i]$

$z^{\lambda}_{i,\ell}$ $= 1$ if activity $i \in \mathscr{A}$ starts in period $\ell \in \mathscr{L}$ or after, 0 otherwise

$z^{\mu}_{i,\ell}$ $= 1$ if activity $i \in \mathscr{A}$ completes in period $\ell \in \mathscr{L}$ or before, 0 otherwise

$\lambda_{i,\ell}$ Length of $[(\ell-1)\Delta, \ell\Delta] \cap [0, S_i]$

$\mu_{i,\ell}$ Length of $[(\ell-1)\Delta, \ell\Delta] \cap [S_i + p_i, L\Delta]$



i
S_i
1 2 3 4 5 6 7 8 9 10 11
ℓ
Δ
0
$d_{i,\ell}$
1
0
$z^{\lambda}_{i,\ell}$
1
0
$z^{\mu}_{i,\ell}$
Δ
0
$\lambda_{i,\ell}$
Δ
0
$\mu_{i,\ell}$

Period partition

$$\lambda_{i,\ell} + d_{i,\ell} + \mu_{i,\ell} = \Delta \qquad \forall i \in \mathcal{A}\ ,\ \forall \ell \in \mathcal{L}$$

Interdependent step behavior

$$z^{\lambda}_{i,\ell+1} \leq \frac{\lambda_{i,\ell}}{\Delta} \leq z^{\lambda}_{i,\ell} \qquad \forall i \in \mathcal{A}\ ,\ \forall \ell \in \mathcal{L}$$

$$z^{\lambda}_{i,1} = 1 \qquad \forall i \in \mathcal{A}$$

$$z^{\mu}_{i,\ell-1} \leq \frac{\mu_{i,\ell}}{\Delta} \leq z^{\mu}_{i,\ell} \qquad \forall i \in \mathcal{A}\ ,\ \forall \ell \in \mathcal{L}$$

$$z^{\mu}_{i,L} = 1 \qquad \forall i \in \mathcal{A}$$

Computing S_i and $d_{i,\ell}$

$$S_i = \sum_{\ell \in \mathcal{A}} \lambda_{i,\ell} \qquad \forall i \in \mathcal{A}$$

$$\sum_{\ell \in \mathcal{A}} d_{i,\ell} = p_i \qquad \forall i \in \mathcal{A}$$

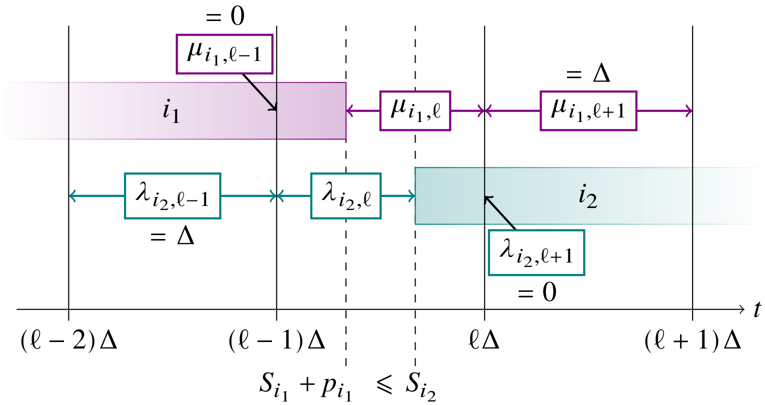
$$d_{i,\ell} \geq 0 \qquad \forall i \in \mathcal{A}\,,\ \forall \ell \in \mathcal{L}$$

Note

$$S_i + p_i = \sum_{\ell \in \mathcal{A}} \lambda_{i,\ell} + \sum_{\ell \in \mathcal{A}} d_{i,\ell} = L\Delta - \sum_{\ell \in \mathcal{A}} \mu_{i,\ell} \qquad \forall i \in \mathcal{A}$$

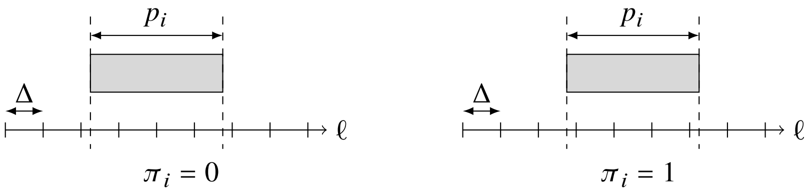
Disaggregation of precedence constraints

$$\mu_{i_1,\ell} + \lambda_{i_2,\ell} \geq \Delta \qquad \forall (i_1, i_2) \in E\,,\ \forall \ell \in \mathscr{L}$$



Linking start and end periods

Two configurations



Link between binary variables

$$\pi_i = 0 \quad \Leftrightarrow \quad z^{\lambda}_{i,\ell} + z^{\mu}_{i,\ell+\left\lfloor \frac{p_i}{\Delta} \right\rfloor - 1} = 1 \qquad \forall i \in \mathcal{A}\,,\ \forall \ell \in \mathcal{L}$$

$$\pi_i = 1 \quad \Leftrightarrow \quad z^{\lambda}_{i,\ell} + z^{\mu}_{i,\ell+\left\lceil \frac{p_i}{\Delta} \right\rceil - 1} = 1 \qquad \forall i \in \mathcal{A}\,,\ \forall \ell \in \mathcal{L}$$

Linearisation

$$z^{\lambda}_{i,\ell} + z^{\mu}_{i,\ell+\lfloor \frac{p_i}{\Delta} \rfloor -1} \leq 1 \qquad \forall i \in \mathcal{A} \,, \; \forall \ell \in \mathcal{L}$$

$$z^{\lambda}_{i,\ell} + z^{\mu}_{i,\ell+\lfloor \frac{p_i}{\Delta} \rfloor -1} \geq 1 - \pi_i \qquad \forall i \in \mathcal{A} \,, \; \forall \ell \in \mathcal{L}$$

$$z^{\lambda}_{i,\ell} + z^{\mu}_{i,\ell+\lceil \frac{p_i}{\Delta} \rceil -1} \geq 1 \qquad \forall i \in \mathcal{A} \,, \; \forall \ell \in \mathcal{L}$$

$$z^{\lambda}_{i,\ell} + z^{\mu}_{i,\ell+\lceil \frac{p_i}{\Delta} \rceil -1} \leq 2 - \pi_i \qquad \forall i \in \mathcal{A} \,, \; \forall \ell \in \mathcal{L}$$

Special case: $p_i \bmod \Delta = 0$

$$z^{\lambda}_{i,\ell} + z^{\mu}_{i,\ell+\frac{p_i}{\Delta}-1} = 1 \qquad \forall i \in \mathcal{A} \,, \; \forall \ell \in \mathcal{L}$$

$$\lambda_{i,\ell} + \mu_{i,\ell+\frac{p_i}{\Delta}} = \Delta \qquad \forall i \in \mathcal{A} \,, \; \forall \ell \in \mathcal{L}$$

Model sizes

Formulation	(MTF-1)	(MTF-2)
# continuous variables	$n + nL$	$2nL$
# binary variables	$2nL$	
# precedence constraints	$\lvert E \rvert$	$\lvert E \rvert L$
# aggregated resource constraints	mL	
# other constraints without $\mathscr{M}$	$n + 4nL$	$n + 5nL$
# other constraints with $\mathscr{M}$	$6nL$	0
# additional binary variables	n	
# additional constraints	$4nL$	

Theorem

Formulation (MTF-1) is not stronger than (MTF-2).

Proof (partial)

$$\begin{aligned} d_{i,\ell} - \Delta\left(1 - z^{\lambda}_{i,\ell} - z^{\mu}_{i,\ell}\right) &= \Delta - \lambda_{i,\ell} - \mu_{i,\ell} - \Delta + \Delta z^{\lambda}_{i,\ell} + \Delta z^{\mu}_{i,\ell} \\ &= \Delta\left(z^{\lambda}_{i,\ell} - \frac{\lambda_{i,\ell}}{\Delta}\right) + \Delta\left(z^{\mu}_{i,\ell} - \frac{\mu_{i,\ell}}{\Delta}\right) \\ &\geq 0 \end{aligned}$$

$$\begin{aligned} d_{i,\ell} - \Delta\left(1 - z^{\lambda}_{i,\ell+1} - z^{\mu}_{i,\ell-1}\right) &= \Delta - \lambda_{i,\ell} - \mu_{i,\ell} - \Delta + \Delta z^{\lambda}_{i,\ell+1} + \Delta z^{\mu}_{i,\ell-1} \\ &= \Delta\left(z^{\lambda}_{i,\ell+1} - \frac{\lambda_{i,\ell}}{\Delta}\right) + \Delta\left(z^{\mu}_{i,\ell-1} - \frac{\mu_{i,\ell}}{\Delta}\right) \\ &\leq 0 \end{aligned}$$

Experiments

- PSPLIB – J30 instances

Δ	$\frac{LB_2^*-LB_1^*}{LB_1^*}$ (%)	$\frac{LB_2-LB_1}{LB_1}$ (%)	$\frac{UB_2-UB_1}{UB_1}$ (%)	$t_1(s)$	$t_2(s)$
5	0.069	+0.073	-0.007	137.18	66.84
4	0.075	+0.094	0.000	258.29	220.96
3	0.350	+0.158	-0.040	362.34	327.34
2	0.919	-0.047	-2.116	381.71	414.60
1	2.184	-0.062	-0.352	316.31	379.35

Theorem

Formulation (MTF-2) is stronger than (MTF-1).

Periodically Aggregated Resource-Constrained Project Scheduling Problem

- Exact models based on mixed time frameworks
- Improvement of the linear relaxation

Perspectives

- Modelling: consolidation of the link between continuous time and time-indexed variables
- Adaptation of energetic reasonning
- Integration into a planning/scheduling scheme