Scheduling instructions on a hierarchical architecture

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Outline of the talk

1. The ST200 Processor
2. The Scheduling Problem
3. Analysis
4. Experimental Validation
5. Conclusion

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The ST200 processor produced by STmicroelectronics, used in “set top box” such as DVD player. It has a not so common architecture.

Interested in scheduling instruction on this processor.

The ST200 processor

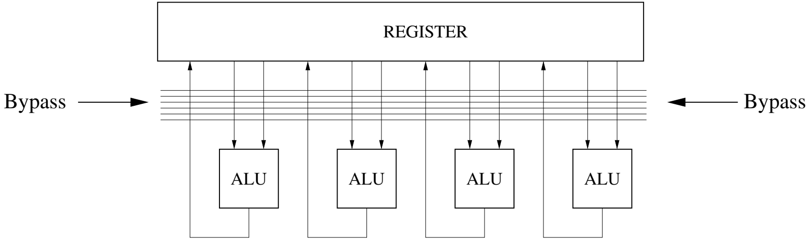

Figure: Current version of ST200

- The result of an operation on an ALU is immediately available on others

The ST200 processor

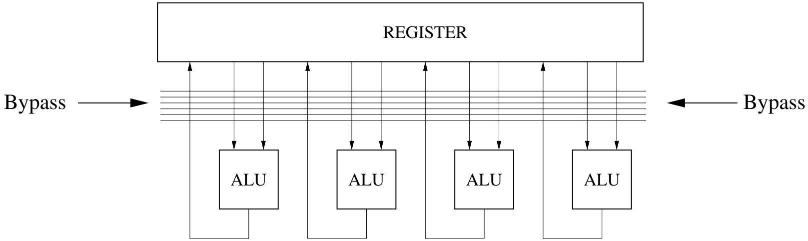

Figure: Current version of ST200

- The result of an operation on an ALU is immediately available on others
- The cost in silicon increases in the square of the number of ALU

The ST200 processor with Incomplete Bypass

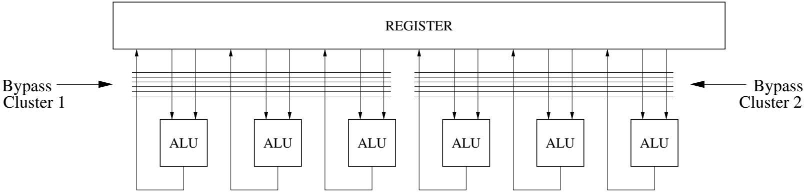

Figure: Future revision of the ST200 processor using an Incomplete Bypass

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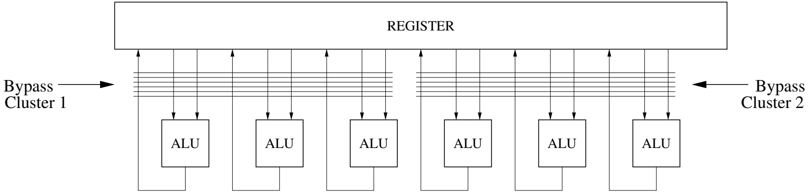

Figure: Future revision of the ST200 processor using an Incomplete Bypass

- The result of an operation on one ALU is immediately available on ALUs of the same cluster, but 2 time clocks later on other clusters
- The cost in silicon increases in the square of the number of ALU in a cluster and linearly in the number of clusters

A compiler problem

How to compile a code for these architectures ?
Mainly 2 problems:

- register allocation
- **instruction scheduling**

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Remark

On complete bypass system, the problem is $P_m \mid prec, p_j = 1 \mid C_{\max}$.
On incomplete bypass ?

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The model

- DAG $G = (T, E)$ where T is a set of n unitary tasks.
- Processors are organized in M clusters of m processors. The l-th cluster is H_l.
- Solution : $\pi : T \rightarrow P$ and $\sigma : T \rightarrow \mathbb{N}^+$
- Between H_i and H_j ($i \neq j$), ρ time units of delay
- Min $C_{\max}$

The problem is denoted by $P_M(P_m)|prec, p_j = 1, c = (\rho, 0)|C_{\max}$ [BGK03]

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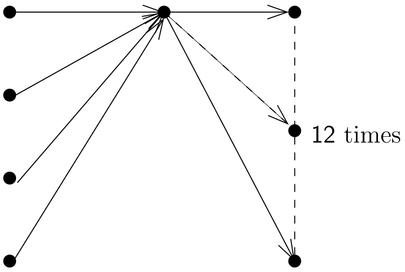
Remark

The $\textsc{st200}$ case is $m = 3$, $M = 2$, $\rho = 2$.

An Example



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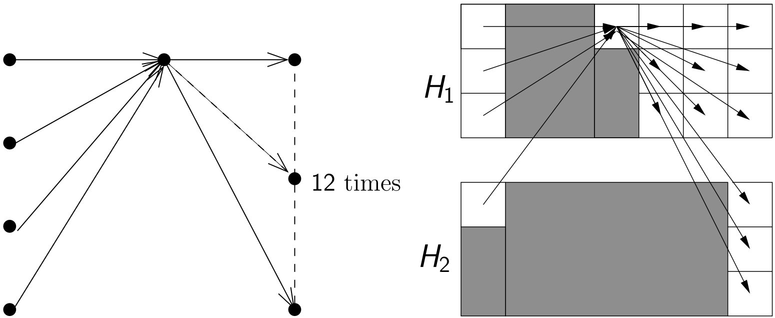

Related works

Complexity

$P_M(P_m)|prec, p_j = 1, c = (\rho, 0)|C_{\max}$ is NP-hard.
The complexity of the $\mathrm{ST200}$ case is not that obvious. It is at least as hard as $P3 \mid prec, p_j = 1 \mid C_{\max}$ which is known to be an open problem.

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Approximability

$P_2(P) \mid bipartite, p_j = 1, c = (1, 0) \mid C_{\max} = 3$ is NP-complete $\Rightarrow$ no approximation algorithm with a performance ratio better than $4/3$ [ABG02].

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List Scheduling with communication has a performance ratio of $2 - \frac{1}{mM} + \rho$

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Definition

An idle at t is an lateness idle if there exists a task released at t scheduled after t.

A nice property

Proposition

A schedule without communicational idle and lateness idle on at least one cluster is $M + 1 - \frac{1}{m}$ *optimal.*

Proof.

sketch:

Two lower bounds. $\frac{n}{Mm}$ (work) and t_∞ (critical path).
Such a schedule have $C_{\max} \leq \frac{n}{m} + t_\infty$.
Thus $C_{\max} \leq MC^*_{\max} + C^*_{\max}$. □

GSingle

Algo

Use List Scheduling on one cluster only.

Corollary

GSingle generates schedules without communicational and lateness idle. Thus it is $M + 1 - \frac{1}{m}$ optimal. In the ST200 *case ($M = 2$ and $m = 3$), GSingle is $\frac{8}{3}$ optimal. (better than LS which is $\frac{23}{6}$)*

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Remark

It uses only $\frac{1}{M}$ of the computational power.

Favorite Cluster

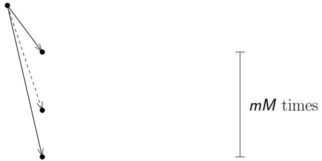
Principle

Let H_1 be the master cluster. Use List scheduling on H_1.
On other clusters H_i. Schedule a task on H_i only if it will be available on H_1 the next time.
If H_1 has a communicational idle, export the last task from H_i to H_1.

Bound

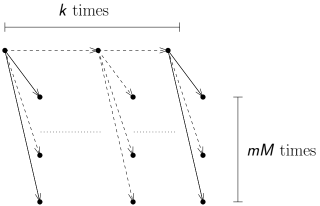
Favorite Cluster generates schedules without communicational and lateness idle. It is a $M + 1 - \frac{1}{m}$-approximation algorithm and the bound is tight.

Tightness



(a)DAG

Tightness



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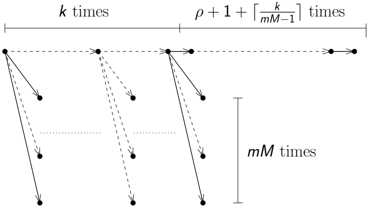

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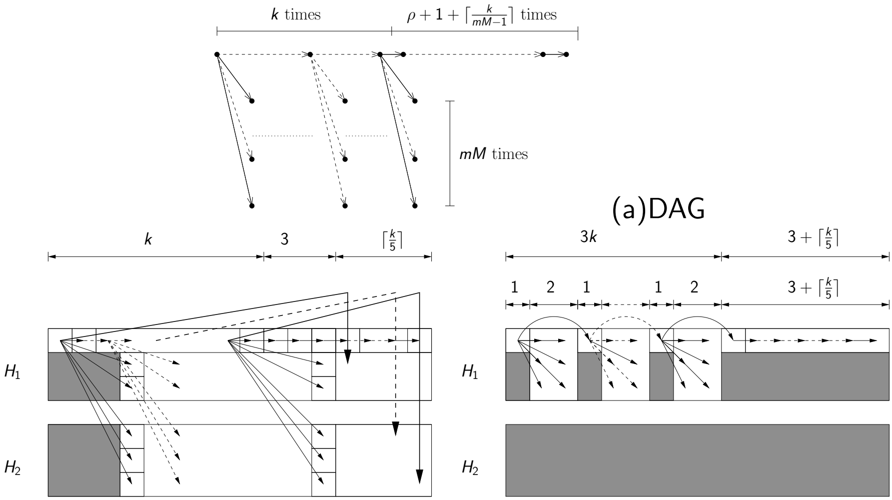


(a)DAG

(b)Optimal schedule

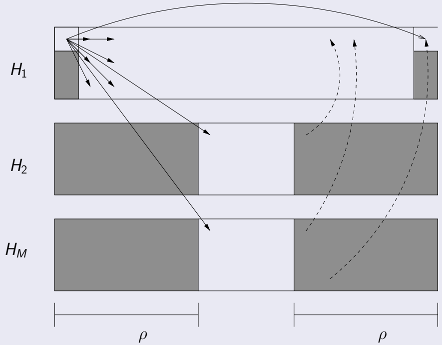
(c)Favorite Cluster schedule

Another Approximation Ratio

Theorem

Favorite Cluster is a $2 + 2\rho - \frac{2\rho}{M} - \frac{1}{Mm}$-approximation algorithm and the bound is tight.

Proof idea



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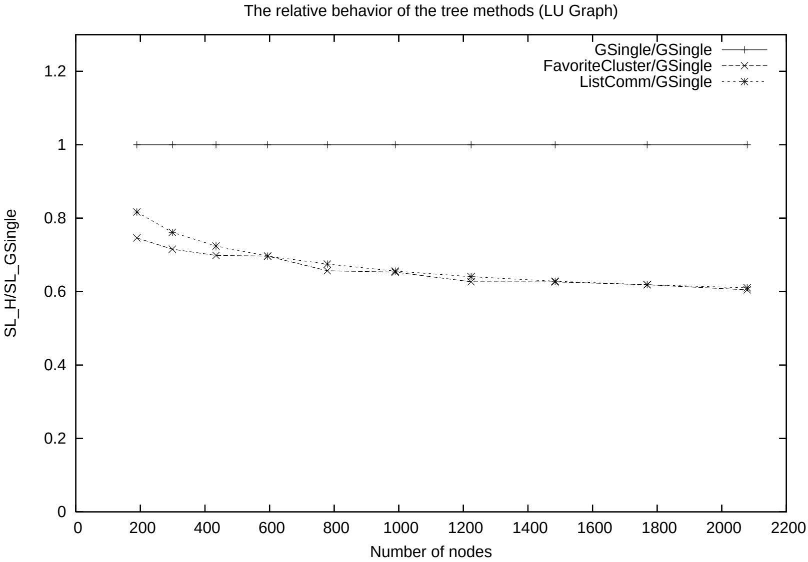
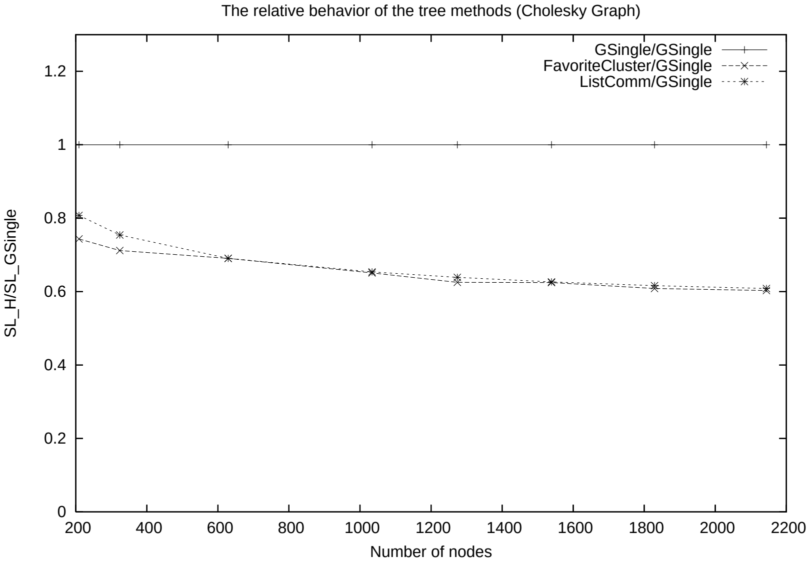
Experimental validation

Goal: compare GSingle, Favorite Cluster and List Scheduling.
From [KA98], benchmarks for $P \mid prec \mid C_{\max}$. Contains randomly generated graphs and **graphs extracted from a parallel compiler**.
On Random graphs: Layered graphs.

Structured Graphs(LU)



Structured Graphs(Cholesky)



Layered Graphs

$$Z = C_{\max}^{FavoriteCluster} - C_{\max}^{LS}$$

Size	30	40	50	60	70	80	90	100	110
$Z < 0$	107	138	198	210	214	219	243	154	239
$Z > 0$	42	52	69	94	103	114	106	89	116
$Z = 0$	351	310	233	196	183	167	151	102	145

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$E[Z]$	-0,232	-0,336	-0,602	-0,654	-0,794	-0,784	-1,036	-0,8841	-0,974
$\sigma[Z]$	0,9433	1,1343	1,6875	1,9187	2,2513	2,4314	2,9053	2,6474	2,7896
$min(Z)$	-5	-6	-10	-9	-11	-11	-16	-11	-15
$max(Z)$	3	2	3	5	4	6	5	4	7
$E[Z] \leq$	-0.1251	-0.2180	-0.4265	-0.4544	-0.5598	-0.5311	-0.7338	-0.6087	-0.6838
$E[C_{\max}^{FavoriteCluster}]$	10,554	13,422	15,712	17,61	19,304	21,706	23,108	25,3188	26,822

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Conclusion

- Present a scheduling problem from the compiler community
- Define different Idle time
- Generalize List Scheduling for $P_M(P_m)|prec, p_j = 1, c = (\rho, 0)|C_{\max}$
- Propose a heuristic with good behavior in practice

Perspective

- Derive a better approximation algorithm (that grows with M)
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- ... or find some inapproximability bounds.
- FavoriteCluster applies to cluster scheduling. Investigate it.

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